Pre-Calculus Name________________________________

Addition and Subtraction Formulas Notes

Date________________ Period____

Use the angle sum identity to find the exact value of each.

1) sin 195°

$\sin(150^\circ + 45^\circ)$

$\sin 150^\circ \cos 45^\circ + \sin 45^\circ \cdot \cos 150^\circ$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

2) cos 75°

$\cos(45^\circ + 30^\circ)$

$\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

Use the angle difference identity to find the exact value of each.

3) $\tan \frac{17\pi}{12}$ $\quad 4\boxed{5}\frac{\pi}{3} - \boxed{1}\frac{\pi}{4} \cdot \frac{3}{3}$

$\tan\left(\frac{5\pi}{3} - \frac{\pi}{4}\right)$

$= \frac{\tan\frac{5\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$

$\frac{-\sqrt{3} - 1}{1 + (-\sqrt{3})(1)}$

$\frac{(-\sqrt{3}-1)}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{-\sqrt{3} - 3 - 1 - \sqrt{3}}{1 - 3}$

$= \frac{-2\sqrt{3}-4}{-2} = \sqrt{3}+2$

4) $\cos -\frac{13\pi}{12}$ $\quad \frac{4}{4}\boxed{4}\frac{\pi}{3} - \frac{\pi}{4} \quad \frac{3}{3}\boxed{1} \rightarrow \frac{-13\pi}{12}$

$\cos\left(\frac{\pi}{4} - \frac{4\pi}{3}\right)$

$\cos\frac{\pi}{4}\cos\frac{4\pi}{3} + \sin\frac{\pi}{4}\sin\frac{4\pi}{3}$

$\frac{\sqrt{2}}{2} \cdot -\frac{1}{2} + \frac{\sqrt{2}}{2} \cdot -\frac{\sqrt{3}}{2}$

$\boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

Use an addition or subtraction formula to write the expression as a trigonometric function of one number, and then find its exact value.

5) $\cos 10° \cos 80° - \sin 10° \sin 80°$

$\cos(10° + 80°)$

$\cos(90°)$

$\boxed{= 0}$

6) $\sin 23° \cos 37° + \sin 37° \cos 23°$

$\sin(23° + 37°)$

$\sin(60°)$

$= \frac{\sqrt{3}}{2}$

7) $\cos \frac{19\pi}{12} \cos \frac{\pi}{4} + \sin \frac{19\pi}{12} \sin \frac{\pi}{4}$

$\cos\left(\frac{19\pi}{12} - \frac{\pi}{4}\frac{3}{3}\right)$

$\cos\left(\frac{19\pi - 3\pi}{12}\right)$

$\cos\left(\frac{16\pi}{12}\right) \to \cos\left(\frac{4\pi}{3}\right) \to -\frac{1}{2}$

8) $(\tan \pi + \tan \frac{\pi}{6}) \div (1 - \tan \pi \tan \frac{\pi}{6})$

$\tan\left(\frac{6}{6}\pi + \frac{\pi}{6}\right)$

$\tan\left(\frac{7\pi}{6}\right) = \frac{\sqrt{3}}{3}$

Prove the identity.

9) $\cos(\theta - \frac{\pi}{2}) = \sin \theta$

$\cos\theta \cdot \cos\frac{\pi}{2} + \sin\theta \sin\frac{\pi}{2}$

$\cos\theta \; 0 + \sin\theta \; 1$

$\sin\theta = \sin\theta$ ✓

10) $\sin(\frac{\pi}{2} - \theta) = \cos \theta$

$\sin\frac{\pi}{2}\cos\theta - \sin\theta \cdot \cos\frac{\pi}{2}$

$\cos\theta - 0$

$\boxed{\cos\theta}$

11) $\cot(x+y) = (\cot x \cot y - 1) \div (\cot x + \cot y)$

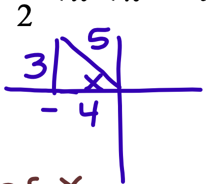
$$\frac{1}{\tan(x+y)}$$

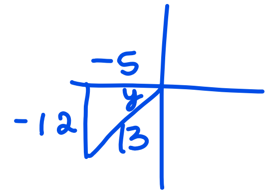
$$\frac{1 - \tan x \tan y}{\tan x + \tan y}$$

$$\frac{\frac{\cot x \cot y}{\cot x \cot y} - \frac{1}{\cot x \cot y}}{\frac{\cot y}{\cot y}\frac{1}{\cot x} + \frac{1}{\cot y}\frac{\cot x}{\cot x}}$$

$$\boxed{= \frac{\cot x \cot y - 1}{\cot y + \cot x}}$$

Assuming $\sin x = \frac{3}{5}$ **if** $\frac{\pi}{2} < x < \pi$ **and** $\cos y = -\frac{5}{13}$ **if** $\pi < y < \frac{3\pi}{2}$





12) $\sin(x + y)$

$\sin x \cos y + \sin y \cos x$

$\left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{-12}{13}\right)\left(\frac{-4}{5}\right)$

$\frac{-15}{65} + \frac{48}{65} = \frac{33}{65}$

13) $\cos(x + y)$

$\cos x \cos y - \sin x \sin y$

$\left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right)$

$\frac{20}{65} + \frac{36}{65} = \frac{56}{65}$

14) $\tan(x - y)$

$\frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\frac{\frac{5}{5}\left(\frac{-3}{4} - \frac{12}{5}\right)\frac{4}{4}}{\frac{20}{20}\left(1 + \left(\frac{-3}{4}\right)\left(\frac{12}{5}\right)\right)}$

$= \frac{-15 - 48}{20 - 36} = \frac{-63}{-16} = \frac{63}{16}$

15) $\sec(x - y)$

Find $\cos(x-y)$

$\cos x \cos y + \sin x \sin y$

$\left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right)$

$\frac{20}{65} - \frac{36}{65} = \frac{-16}{65}$

SO $\boxed{\sec(x-y) = \frac{-65}{16}}$