

Algebra Review Day 1

Simplify.

1) $5\sqrt[3]{(64xy^3)^2 \cdot x^3y^3}$

$5\sqrt[3]{(2^6x^2y^3)^2 x^3y^3}$

$5\sqrt[3]{2^2x^2y^3 \cdot 2^2x^2y^3 \cdot x^3y^3}$

$5 \cdot 2^4 \cdot x^4 \cdot y^3 \cdot \sqrt[3]{x^2}$

$80xy^3\sqrt[3]{x^2}$

$\sqrt[3]{x^2} = x^{\frac{2}{3}}$

Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.

2) $\frac{(u^{\frac{2}{3}}v^{\frac{3}{2}})^{\frac{1}{2}}}{vu^{\frac{4}{3}} \cdot v^{-1}}$

$\frac{u^{\frac{1}{3}}v^{\frac{3}{4}}}{u^{\frac{4}{3}}}$

$u^{\frac{1}{3}-\frac{4}{3}}v^{\frac{3}{4}} = u^{-1}\frac{v^{\frac{3}{4}}}{u}$ OR $\frac{v^{\frac{3}{4}}}{u}$

$v^1 \cdot v^{-1} = 1$ or $v \cdot v^{-1} = \frac{v}{v} = 1$

$v^{1+1} = v^0$

Factor each completely.

3) $4n^3 - 12n^2 - n + 3$

$4n^2(n-3) - 1(n-3)$

$(n-3)(4n^2 - 1) \rightarrow (n-3)(2n-1)(2n+1)$

Difference of squares

Factor each.

$(-2)^5 + 2(-2)^4 - 10(-2)^3 - 20(-2)^2 + 16(-2) + 32$

4) $f(x) = x^5 + 2x^4 - 10x^3 - 20x^2 + 16x + 32$

*rational roots are $\pm 1, 2, 4, 8, 16, 32$

so $x = -2$ is a root

-2	1	2	-10	-20	16	32
		-2	0	20	0	-32
	1	0	-10	0	16	0

$(x+2)(x^4 - 10x^2 + 16)$

$(x+2)(x^2 - 8)(x^2 - 2)$

if $f(x) = 0$

$x+2=0$ $x^2-8=0$ $x^2-2=0$

$x = -2$

$x^2 = 8$ $x^2 = 2$

$x = \pm\sqrt{8}$ or $-\sqrt{8}$ $x = \pm\sqrt{2}$

$$5) f(x) = 27x^5 - 45x^4 + 18x^3 - 30x^2 + 3x - 5$$

*rational root possibilities

$$\pm 5, 1, \pm \frac{5}{3}, \pm \frac{5}{9}, \pm \frac{5}{27}$$

$$\left(x - \frac{5}{3}\right)(27x^4 + 18x^2 + 3)$$

$$(3x - 5)(9x^4 + 6x^2 + 1)$$

$$(3x - 5)(3x^2 + 1)^2$$

$$\begin{array}{r|rrrrrr} \frac{5}{3} & 27 & -45 & 18 & -30 & 3 & -5 \\ & & 45 & 0 & 30 & 0 & 5 \\ \hline & 27 & 0 & 18 & 0 & 3 & 0 \end{array}$$

Simplify each expression.

$$6) \frac{2}{5r^2 - 5r} - \frac{3}{r^2 - 1} + \frac{r}{2r^2 + r - 1}$$

$$\frac{(r+1)(2r-1)2}{5r(r-1)} - \frac{5r \cdot 3(2r-1)}{(r+1)(r-1)5r(2r-1)} + \frac{r \cdot 5r(r-1)}{(2r-1)(r+1)(5r)(r-1)}$$

$$\rightarrow 4r^2 + 2r - 2 - 30r^2 + 15r + 5r^3 - 5r^2$$

$$\rightarrow \frac{5r^3 - 31r^2 + 17r - 2}{5r(2r-1)(r+1)(r-1)}$$

Simplify.

$$7) \frac{\sqrt{2}}{\sqrt{5} - \sqrt{2}} \cdot \frac{(\sqrt{5} + \sqrt{2})}{(\sqrt{5} + \sqrt{2})} \rightarrow \frac{\sqrt{10} + 2}{5 - 2} = \frac{\sqrt{10} + 2}{3}$$

$$x \neq 0, 5$$

Solve each equation. Remember to check for extraneous solutions.

$$8) \frac{x+2}{x} = \frac{x+4}{x} - \frac{5x-30}{x^2-5x}$$

$$\frac{x+2}{x} = \frac{x+4}{x} - \frac{5x-30}{x(x-5)}$$

$$\frac{x+2-x-4}{x} = \frac{-5x+30}{x(x-5)}$$

$$\frac{-2}{x} = \frac{-5x+30}{x(x-5)}$$

$$-2x(x-5) = x(-5x+30)$$

$$-2x^2 + 10x = -5x^2 + 30x$$

$$3x^2 - 20x = 0$$

$$x(3x-20) = 0$$

$$x = 0, \frac{20}{3}$$

Extraneous

Solve each inequality.

$$9) |r-7| \leq 1$$

$$\begin{array}{r} r-7 \leq 1 \\ +7 \quad +7 \\ \hline r \leq 8 \end{array} \quad \begin{array}{r} r-7 \geq -1 \\ +7 \quad +7 \\ \hline r \geq 6 \end{array}$$

$$6 \leq r \leq 8$$

$$10) |-5n| > 45$$

$$\begin{array}{r} -5n > 45 \\ -5n < -45 \end{array}$$

$$n < -9 \quad \text{or} \quad n > 9$$

Write the slope-intercept form of the equation of the line through the given point with the given slope.

$$y = mx + b$$

$$11) \text{ through: } (-4, 3), \text{ slope} = -\frac{5}{4}$$

$$\text{Point-slope form } y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{5}{4}(x + 4)$$

$$y = -\frac{5}{4}x - 5 + 3$$

$$y = -\frac{5}{4}x - 2$$

Write the slope-intercept form of the equation of the line through the given points.

$$12) \text{ through: } (1, -3) \text{ and } (4, 1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 + 3}{4 - 1} = \frac{4}{3}$$

$$\textcircled{1} y = \frac{4}{3}x - \frac{4}{3} - 3 \frac{3}{3}$$

$$\textcircled{2} y = \frac{4}{3}x - \frac{16}{3} + 1 \frac{3}{3}$$

one option

2nd option

$$y + 3 = \frac{4}{3}(x - 1)$$

$$y - 1 = \frac{4}{3}(x - 4)$$

$$y = \frac{4}{3}x - \frac{13}{3}$$

Write the slope-intercept form of the equation of the line described.

$\perp \rightarrow$ opposite, reciprocal

13) through: (3, -4), parallel to $y = -2x - 3$
same slope

$$m = -2$$

$$y + 4 = -2(x - 3)$$

14) through: (3, 0), perp. to $6x + 4y - 4 = 0$

$$\frac{4y}{4} = \frac{-6x + 4}{4}$$

$$y = -\frac{3}{2}x + 1 \quad \perp \text{ slope} = \frac{2}{3}$$

$$y = \frac{2}{3}(x - 3)$$

15) Assume that M varies directly as z , and $M = 126$ if $z = 14$. Write an equation that expresses this variation.

Direct

Inverse

$$y = kx$$

$$k = \frac{xy}{x}$$

where k is a constant

$$M = kz$$

$$126 = k \cdot 14$$

$$k = 9$$

$$M = 9z$$

16) In a certain city, the property tax collected for a home varies directly to the valuation of the property. The tax collected on a \$105,000 home is \$2,846 per year. What is the value of a home if the tax collected is \$1,735?

$$\frac{2,846}{105,000} = \frac{1,735}{x}$$

$$x = \$64,010.89$$

$$2846x = 182,175,000$$

17) The resistance of a wire varies directly as its length and inversely as the square of its diameter. A wire 50 m long and 1 cm in diameter has a resistance of 25 ohms. Find the resistance of a wire made of the same material that is 20 m long and has diameter 20 mm.

$$\text{cm} \rightarrow 50\text{m} = 5000\text{cm}$$

$$1\text{cm}$$

$$20\text{m} = 2000\text{cm}$$

$$20\text{mm} = 2\text{cm}$$

$$R = \frac{1}{200} \frac{L}{d^2} \rightarrow R = \frac{1}{200} \frac{(2000)}{2^2}$$

$$R = 2.5 \text{ ohms}$$

$$R = \frac{kL}{d^2}$$

$$k = \frac{1}{200}$$

$$k = \frac{25}{250000}$$

$$25 = \frac{k \cdot 5000}{1}$$