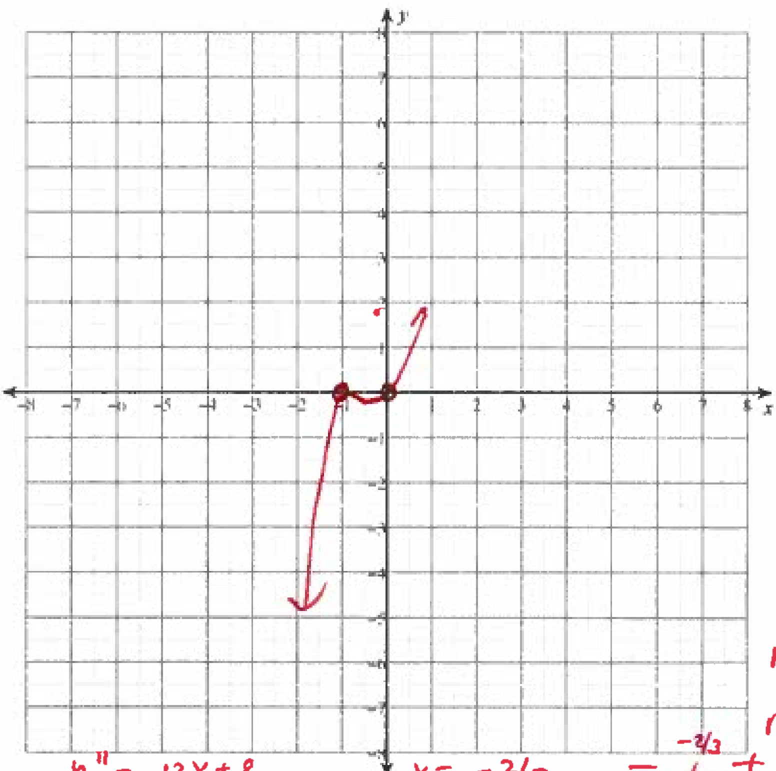
Application of Derivatives Practice Name Key

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = 2x^3 + 4x^2 + 2x$



$y = x(2x^2 + 4x + 2)$

$y = 2x(x^2 + 2x + 1)$

$y = 2x(x+1)^2$

x-int: 0, −1

$y' = 6x^2 + 8x + 2$

$y' = 2(3x^2 + 4x + 1)$

$= 2(3x+1)(x+1)$

$x = -\frac{1}{3}, -1$ + | − | + (−1, −1/3)

rel max @ −1 → 0

rel min @ −1/3 → −8/27

$y'' = 12x + 8$ $x = -2/3$ ccd − | + ccu (−2/3)

For each problem, find all points of relative minima and maxima.

2) $f(x) = \dfrac{x^2}{4x - 8}$

$\dfrac{4(x-2)(2x) - x^2(4)}{(4x-8)^2}$

$\dfrac{(4x-8)(2x) - x^2(4)}{(4x-8)^2}$

CN: 0, 4, 2

$\dfrac{4(2x^2 - 4x - x^2)}{4 \cdot 4(x-2)^2} \rightarrow \dfrac{x(x-4)}{4(x-2)^2}$

+ | − | − | + (0, 2, 4)

rel max = 0 @ x = 0

rel min = 2 @ x = 4

3) $y = -2\sin(2x);\ [-\pi, \pi]$

$y' = -2\cos(2x) \cdot 2$

$\cos 2x = 0$ @ $\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$

$2x =$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, -\frac{\pi}{4}, -\frac{3\pi}{4}$

[− | + | − | + | −] ($-\pi$, $-3\pi/4$, $-\pi/4$, $\pi/4$, $3\pi/4$, π)

rel min = −2 @ $x = -\frac{3\pi}{4}, \frac{\pi}{4}$

rel. max = 2 @ $x = -\frac{\pi}{4}, \frac{3\pi}{4}$

For each problem, find all points of absolute minima and maxima on the given interval.

4) $y = x^3 - 3x^2 - 1;\ [-1, 1]$

$y' = 3x^2 - 6x$

$y' = 3x(x-2)$ 0, 2

| + | − | ~~2~~ (−1, 0, 1)

abs max = −1 @ x = 0

abs min = −5 @ x = −1

5) $y = -x^4 + 2x^2 - 1;\ [-1, 1]$

$y' = -4x^3 + 4x$

$y' = -4x(x^2 - 1)$

$-4x(x+1)(x-1)$

| − | + | (−1, 0, 1)

abs min = −1 @ x=0

abs max = 0 @ x=±1

$$\left[\left(\frac{-(-1)^2+1}{-2}\right)-\left(\frac{-(-4)^2+1}{-8}\right)\right] \div [-1--4] = -\frac{5}{8}$$

For each problem, find the values of c that satisfy the Mean Value Theorem.

6) $y = \dfrac{-x^2+1}{2x}$; $[-4, -1]$

$$\frac{2x(-2x)-(-x^2+1)(2)}{4x^2}$$

$$\frac{-4x^2+2x^2-2}{4x^2}$$

$$\frac{-2x^2-2}{4x^2} = \frac{-5}{8}$$

$$-16x^2-16+20x^2=0$$

$$4x^2-16=0$$

$$x^2-4=0$$

$$x=\pm 2$$

$$x=-2$$

7) $y = -2x^2 + 12x - 15$; $[1, 5]$

$$y' = -4x+12$$

$$y' = 0$$

$$x = 3$$

For each problem, find the open intervals where the function is increasing and decreasing.

8) $y = \dfrac{x^2}{2x-4}$

$$y' = \frac{(2x-4)(2x)-x^2(2)}{(2x-4)^2}$$

$$\frac{4x^2-8x-2x^2}{2\cdot 2(x-2)^2}$$

$$\frac{2x^2-8x}{}$$

$$\frac{2x(x-4)}{4(x-2)^2}$$

Inc Dec Dec Inc

$+$ | $-$ | $-$ | $+$

0 2 4

9) $y = -x^3 + 2x^2 - 2$

$$y' = -3x^2+4x$$

$$y' = -x(3x-4)$$

Dec Inc Dec

$-$ | $+$ | $-$

0 4/3

For each problem, find the open intervals where the function is concave up and concave down.

10) $y = -2\sin(2x)$; $[-\pi, \pi]$

$$y' = -4\cos(2x) \qquad y'' = 8\sin(2x) = 0$$

$$\sin 2x = 0$$

$$2x = 0, \pm\pi, \pm 2\pi,$$

$$x = 0, \pm\frac{\pi}{2}, \pm\pi$$

CCU CCD CCU CCD

| $+$ | $-$ | $+$ | $-$ |

$-\pi$ $-\frac{\pi}{2}$ 0 $\frac{\pi}{2}$ π

11) $y = x^3 - 4x^2 + 3$

$$y' = 3x^2 - 8x$$

$$y'' = 6x - 8$$

CCD $-$ | $+$ CCU

4/3