AB Calculus Arc Length Homework Name: Key

1. Find the length of the curve $y = 3x^{\frac{3}{2}} - 1$ over the interval $[0, 1]$. $y' = \frac{3}{2}(3)x^{\frac{1}{2}}$

$$L = \int_0^1 \sqrt{1 + \left(\frac{9\sqrt{x}}{2}\right)^2}\, dx \to \text{calc} \to \boxed{3.192}$$

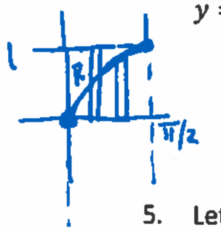
2. Find the length of the curve $y = \ln(\sec x)$ over the interval $\left[0, \frac{\pi}{4}\right]$. $y' = \frac{1}{\sec x}(\sec x \cdot \tan x) = \tan x$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx \to \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx \to \int_0^{\pi/4} \sec x\, dx \to \boxed{.881}$$

3. Find the length of the curve $x = \frac{1}{8}y^4 + \frac{1}{4y^2}$ from $y = 1$ to $y = 4$. $\frac{1}{4}y^{-2}$ $x' = \frac{1}{2}y^3 - \frac{1}{2}y^{-3}$

$$L = \int_1^4 \sqrt{1 + \left(\tfrac{1}{2}y^3 - \tfrac{1}{2}y^{-3}\right)^2}\, dy \to \boxed{32.109}$$

4. Find the volume of the solid generated by revolving the region (R) bounded by the graphs of $y = 1$, $x = 0$, $y = \sin x$ and $x = \frac{\pi}{2}$, about the x-axis.



$$V = \pi \int_0^{\pi/2} \left((1-0)^2 - (\sin x - 0)^2\right) dx \boxed{\to 2.467}$$

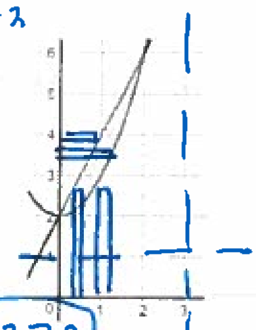
5. Let R be the region bounded by the graphs of $y = x^2 + 2$ and $y = 2x + 2$. $x^2 + 2 = 2x + 2$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$



a) Find the volume of the solid generated by revolving R about the x-axis.

$$V = \pi \int_0^2 \left((2x+2-0)^2 - (x^2+2-0)^2\right) dx \to \boxed{30.159}$$

b) Find the volume of the solid generated by revolving R about the y-axis.

$y - 2 = x^2$
$x = \sqrt{y-2}$

$y = 2x + 2$
$y - 2 = 2x$
$\frac{y-2}{2} = x$

$$V = \pi \int_2^6 \left((\sqrt{y-2} - 0)^2 - \left(\tfrac{y-2}{2} - 0\right)^2\right) dy \to \boxed{8.378}$$

c) Find the volume of the solid generated by revolving R about the line y = 1.

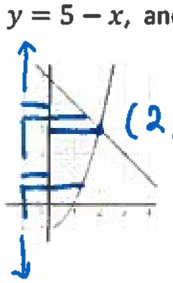
$$V = \pi \int_0^2 \left((2x+2-1)^2 - (x^2+2-1)^2\right) dx \approx \boxed{21.782}$$

d) Find the volume of the solid generated by revolving R about the $x = 3$.

$$V = \pi \int_2^6 \left(\left(3 - \tfrac{y-2}{2}\right)^2 - \left(3 - \sqrt{y-2}\right)^2\right) dy \approx \boxed{16.755}$$

6. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x^2 - 1$, $y = 5 - x$, and the x-axis, about the line $x = -1$.

$x = 5 - y$ $x = \sqrt{y+1}$



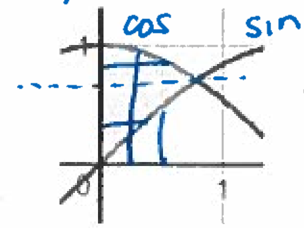
$$V = \pi \int_3^5 \left((5 - y + 1)^2 - (0+1)^2\right) dy$$

$$+ \pi \int_{-1}^3 \left((\sqrt{y+1} + 1)^2 - (0+1)^2\right) dy$$

$$\boxed{\approx 79.587}$$

$y = \sin x \quad \sin^{-1} y = x \quad y = \cos x \quad \cos^{-1} y = x$

7. Let Region R be the shaded region in the first quadrant enclosed by the graphs of $y = \sin x$, $y = \cos x$, and the y-axis as shown in the figure to the right.



a) Find the area of region R. $\cos x = \sin x$ if $x = \pi/4$ & $y = \frac{\sqrt{2}}{2}$

$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx \approx \boxed{.414}$$

b) Find the volume of the solid generated when R is revolved around the x-axis.

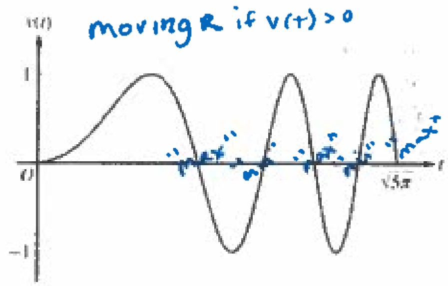
$$V = \pi \int_0^{\pi/4} \left((\cos x - 0)^2 - (\sin x - 0)^2 \right) dx \approx \boxed{1.571}$$

c) Find the volume of the solid generated when R is revolved around the y-axis (shell method). → or in 2 parts like I do here.

$$V = \pi \int_0^{\sqrt{2}/2} \left((\sin^{-1} y)^2 \right) dy + \pi \int_{\sqrt{2}/2}^{1} \left((\cos^{-1} y)^2 \right) dy \approx \boxed{.696}$$

8. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown in the graph to the right for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

moving R if $v(t) > 0$



a) Find the acceleration of the particle at time $t = 3$.

$$a(3) = v'(3) = \boxed{-5.467}$$

b) Find the total distance traveled by the particle from $t = 0$ to $t = 3$.

$$\text{Total Distance Traveled} = \int_0^3 |v(t)|\,dt \approx \boxed{1.702}$$

c) Find the position of the particle at time $t = 3$.

$$x(t) = x(0) + \int_0^3 v(t)\,dt \approx \boxed{5.774}$$

d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

times when particle stops moving right →

$v(1.772) = 0 \quad \rightarrow \int_0^{1.772} v(t)\,dt \approx .895$

$v(3.06998) = 0 \quad \rightarrow \int_0^{3.06998} v(t)\,dt \approx .788$

$v(\sqrt{5\pi}) = 0 \quad \rightarrow \int_0^{\sqrt{5\pi}} v(t)\,dt \approx .752$

farthest right at $t \approx 1.772$ since $x(1.772)$ is the greatest possible value.