AB Calculus Area between Curves Practice **Name:** ______________________________

1. Find the area of the region bounded by the graphs of $f(x) = 1 + 2x - x^2$ and $g(x) = x - 1$.

$1+2x-x^2 = x-1$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2, -1$

$A = \int_{-1}^{2} \left(-x^2+2x+1-(x-1)\right)dx = 4.5$

2. Find the area of the region bounded by the graphs of $f(x) = \frac{4}{2-x}$, $y = 4$, and $x = 0$.

$\frac{4}{1} = \frac{4}{2-x}$

$4(2-x) = 4$

$2-x = 1$

$x = 1$

$\int_0^1 \left(4 - \frac{4}{2-x}\right)dx$

≈ 1.227

3. Find the area of the region bounded by the graphs of $y = 1 - x^2$ and $y = 1 - x$.

$1-x^2 = 1-x$

$-x^2 = -x$

$0 = x^2 - x$

$0 = x(x-1)$

$x = 0$

$x = 1$

$\int_0^1 \left((1-x^2)-(1-x)\right)dx$

$\int_0^1 (-x^2+x)\,dx = -\frac{x^3}{3} + \frac{x^2}{2}\Big|_0^1 = -\frac{1}{3} + \frac{1}{2} = 1/6$

4. Find the area of the shaded region between the graphs of $x = y^3$ and $x = y^2$.

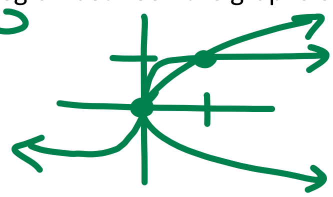
$y^3 = y^2$

$y^3 - y^2 = 0$

$y^2(y-1) = 0$

$y = 0 \rightarrow x = 0$

$y = 1 \rightarrow x = 1$

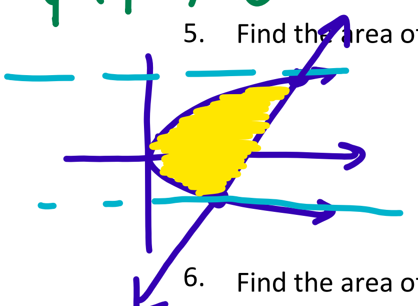


$\int_0^1 (y^2 - y^3)\,dy \rightarrow \frac{y^3}{3} - \frac{y^4}{4}\Big|_0^1$

$\int_0^1 (\sqrt[3]{x} - \sqrt{x})\,dx$

$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

5. Find the area of the shaded region between the graphs of $x = y^2$ and $x = y + 6$.



$y^2 = y + 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

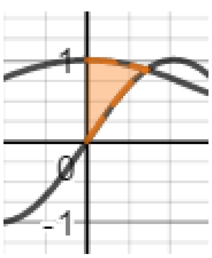
$y = 3 \rightarrow x = 9$

$y = -2 \rightarrow x = 4$

$\int_{-2}^{3} (y+6-y^2)\,dy = 20.833$

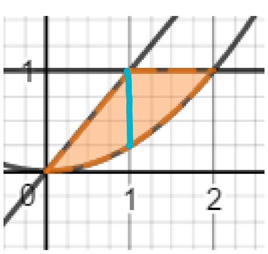
6. Find the area of the region in the first quadrant bounded by the graphs of $f(x) = e^{-x^2}$, $g(x) = \sin 3x$, and the y-axis.

Intersect at (.358, Irre.)



$\int_0^{.358} (f(x) - g(x))\,dx = .169$

7. Find the area of the region bounded by the graphs of $y = x$, $y = 1$, and $y = \frac{x^2}{4}$.



$\int_0^1 (\sqrt{4y} - y)\,dy$

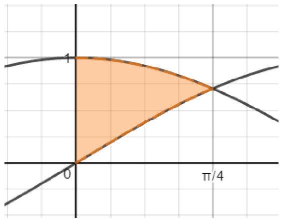
$4y = x^2$

$\sqrt{4y} = x$

$= 5/6$

$\int_0^1 \left(x - \frac{x^2}{4}\right)dx + \int_1^2 \left(1 - \frac{x^2}{4}\right)dx$

8. Find the area of the region bounded by the graphs of $y = \sin x$, $y = \cos x$ and $x = 0$.



$$\int_0^{\pi/4} (\cos x - \sin x)\, dx$$

$$= \left[\sin x + \cos x\right]_0^{\pi/4} = \left(\sin\tfrac{\pi}{4} + \cos\tfrac{\pi}{4}\right) - \left(\sin 0 + \cos 0\right)$$

$$\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2} - 0 - 1 = \boxed{\sqrt{2} - 1}$$

9. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 AM, snow accumulates on the driveway at a rate modeled by the function $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 AM ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0, & 0 \le t < 6 \\ 125, & 6 \le t < 7 \\ 108, & 7 \le t \le 9 \end{cases}$$

a) How many cubic feet of snow have accumulated on the driveway by 6 AM?

$$\int_0^6 7te^{\cos t}\, dt = 142\ 275$$

b) Find the rate of change of the volume of snow on the driveway at 8 AM.

$$f(8) - g(8) = -59\ 583\ \text{ft}^3/\text{hr}$$

c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time for t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

$$h(t) = \begin{cases} 0 & 0 < t < 6 \\ 125(t-6) & 6 \le t < 7 \\ 108(t-7) + 125 & 7 \le t \le 9 \end{cases}$$

d) How many cubic feet of snow are on the driveway at 9 AM?

$$\int_0^9 7te^{\cos t}\, dt - \left(108(2) + 125\right) = 26\ 335$$