AB Calculus Area between Curves Practice Name: ______________________________

1. Find the area of the region bounded by the graphs of $f(x) = 1 + 2x - x^2$ and $g(x) = x - 1$.

$1+2x-x^2 = x-1$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$ $x = 2, -1$

$\int_{-1}^{2} (1+2x-x^2-(x-1))\,dx = \int_{-1}^{2}(-x^2+x+2)\,dx$

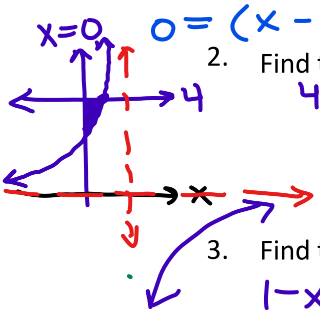
$= -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2}$

$= \left(-\frac{8}{3}+2+4\right) - \left(\frac{1}{3}+\frac{1}{2}-2\right)$

$-3+8-\frac{1}{2}$

$5-\frac{1}{2} = \boxed{4.5}$

2. Find the area of the region bounded by the graphs of $f(x) = \frac{4}{2-x}$, $y = 4$, and $x = 0$.



$4 = \frac{4}{2-x}$

$4(2-x) = 4$

$2-x = 1$

$x = 1$

$\int_0^1 \left(4 - \frac{4}{2-x}\right)dx = 4x - \frac{4\ln|2-x|}{-1}\Big|_0^1 = (4+4\ln 1) - (0+4\ln 2)$

$= \boxed{4-4\ln 2}$

3. Find the area of the region bounded by the graphs of $y = 1 - x^2$ and $y = 1 - x$.

$1-x^2 = 1-x$

$-x^2 = -x$

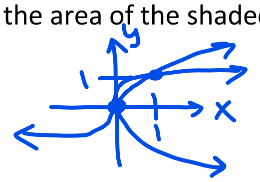
$0 = x^2 - x$

$0 = x(x-1) \rightarrow x = 0, 1$

$\int_0^1 ((1-x^2)-(1-x))\,dx$

$\int_0^1 (-x^2+x)\,dx = -\frac{x^3}{3} + \frac{x^2}{2}\Big|_0^1 = -\frac{1}{3}+\frac{1}{2} = \frac{1}{6}$

4. Find the area of the shaded region between the graphs of $x = y^3$ and $x = y^2$.



$y^3 = y^2$

$y^3 - y^2 = 0$

$y^2(y-1) = 0$ $y = 0, 1$

$x = y^3$ $x = y^2$

$\sqrt[3]{x} = y$ $\sqrt{x} = y$

$y \rightarrow 1$, $y \rightarrow 0$: $\int_0^1 (y^2 - y^3)\,dy \rightarrow \frac{y^3}{3} - \frac{y^4}{4}\Big|_0^1 \rightarrow \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

$x \rightarrow 1$, $x \rightarrow 0$: $\int_0^1 (\sqrt[3]{x} - \sqrt{x})\,dx \rightarrow \frac{3x^{4/3}}{4} - \frac{2x^{3/2}}{3}\Big|_0^1 = \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$

5. Find the area of the shaded region between the graphs of $x = y^2$ and $x = y + 6$.

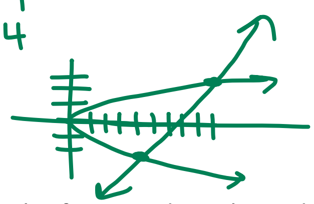
$y^2 = y+6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) \rightarrow 3, -2$

$x = 3+6 = 9$

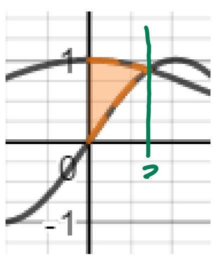
$x = -2+6 = 4$



$\int_{-2}^{3} (y+6-y^2)\,dy$

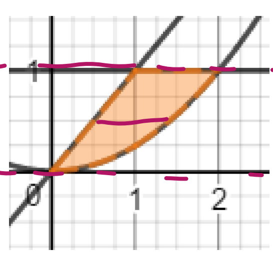
$= \frac{y^2}{2} + 6y - \frac{y^3}{3}\Big|_{-2}^{3} = \frac{125}{6}$

6. Find the area of the region in the first quadrant bounded by the graphs of $f(x) = e^{-x^2}$, $g(x) = \sin 3x$, and the y-axis.



$\int_0^{3583} \left(e^{-x^2} - \sin(3x)\right) = 169$

7. Find the area of the region bounded by the graphs of $y = x$, $y = 1$, and $y = \frac{x^2}{4}$.



$y = \frac{x^2}{4}$

$4y = x^2$

$\sqrt{4y} = x$

$y = x$

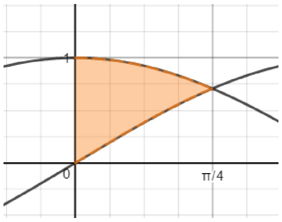
$x = y$

$\int_0^1 (\sqrt{4y} - y)\,dy$

*this can go into calculator w/ "x" as the variable

$\int_0^1 (2\sqrt{y} - y)\,dy = 2\,\frac{2}{3}y^{\frac{3}{2}} - \frac{y^2}{2}\Big]_0^1 = \frac{4}{3} - \frac{1}{2} = \frac{8}{6} - \frac{3}{6} = 5/6$

8. Find the area of the region bounded by the graphs of $y = \sin x$, $y = \cos x$ and $x = 0$.



$$\int_0^{\pi/4} (\cos x - \sin x)\,dx = \sin x + \cos x \Big|_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1$$

$$\boxed{= \sqrt{2} - 1}$$

9. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 AM, snow accumulates on the driveway at a rate modeled by the function $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 AM ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0, & 0 \le t < 6 \\ 125, & 6 \le t < 7 \\ 108, & 7 \le t \le 9 \end{cases}$$

Amt of snow at midnight

a) How many cubic feet of snow have accumulated on the driveway by 6 AM?

$$0 + \int_0^6 f(t)\,dt =$$

b) Find the rate of change of the volume of snow on the driveway at 8 AM.

c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time for t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

d) How many cubic feet of snow are on the driveway at 9 AM?