BC Calculus Vector-Valued Functions **Name:** ______________________________

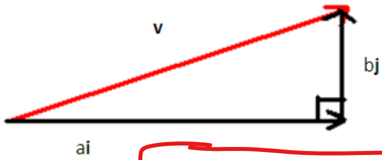
$(1,0)$

$(0,1)$

Linear Combination Form

Any vector $\mathbf{v} = \langle a, b \rangle$ can be written as a linear combination of the two **standard unit vectors.**

$$\mathbf{i} = \langle 1,0 \rangle \qquad \mathbf{j} = \langle 0,1 \rangle$$

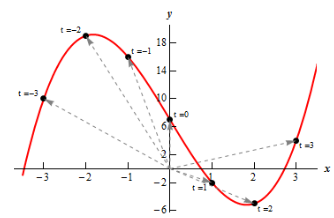


The vector **v** is a **linear combination** of the vectors **i** and **j**. The scalar *a* is the **horizontal component** of **v** and the scalar *b* is the **vertical component** of **v.**

Example 1 Let $P = (-1,5)$ and $Q = (3,2)$. Write the vector $\overrightarrow{PQ}$ as a linear combination of **i** and **j**.

$PQ = \langle 4, -3 \rangle$ or $4i - 3j$

Vector-Valued Functions



A particle moves through the plane on a time interval *I* with the coordinates of the particle as functions defined by *I*: $x = f(t)$ $y = g(t)$

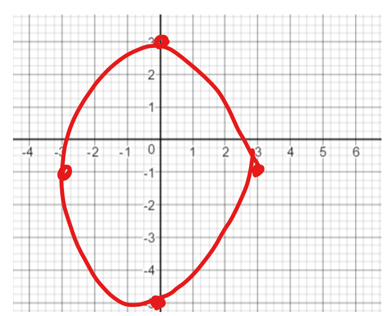
The points $(x, y) = \left(f(t), g(t)\right), t \in I$ make up the curve in the plane that is the particle's path.

The vector $\mathbf{r}(t) = \overrightarrow{OP} = \langle f(t), g(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j}$ from the origin to the particle's position at time t is the **position vector.** The functions *f* and *g* are the **component functions (components)** of the position vector. The particle's path is the curve traced by r during the time interval *I*.

Example 2 Graph the vector function

$r(t) = (3\cos t)\mathbf{i} + (4\sin t - 1)j$ where $t > 0$.

t	$3\cos t$	$4\sin t - 1$
$\pi/2$	0	3
π	-3	-1
$3\pi/2$	0	-5
2π	3	-1



Limit of a vector function

Let $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$. If $\lim_{t \to c} f(t) = L_1$ and $\lim_{t \to c} g(t) = L_2$, then the limit of $r(t)$ as t approaches c is $\lim_{t \to c} \mathbf{r}(t) = \boldsymbol{L} = L_1\mathbf{i} + L_2\mathbf{j}$.

Example 3 Let $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$. Find $\lim_{t \to \pi/4} \mathbf{r}(t)$.

$$\lim_{t \to \pi/4} r(t) = \left(\cos\frac{\pi}{4}\right)i + \left(\sin\frac{\pi}{4}\right)j = \frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$$

Example 4 Let $\mathbf{r}(t) = (t^2)\mathbf{i} + \ln(t-3)\,\mathbf{j}$. Find $\lim_{t \to 0} \mathbf{r}(t)$.

$$\lim_{t \to 0} r(t) = (0^2)i + \ln(-3)j$$

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Derivative of a vector function

Let $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ with components made up of the differentiable functions $f(t)$ and $g(t)$. Then, the derivative of $\mathbf{r}(t)$ is

$$\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t) = f'(t)\mathbf{i} + g'(t)\mathbf{j}$$

Example 5 Let $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$. Find $\mathbf{r}'(t)$.

$r'(t) = (-\sin t)i + (\cos t)j$

Integral of a vector function

Let $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ with components made up of the integrable functions $f(t)$ and $g(t)$. Then, the definite integral of $\mathbf{r}(t)$ from a to b is

$$\int_a^b \mathbf{r}(t)dt = \left(\int_a^b f(t)dt\right)\mathbf{i} + \left(\int_a^b g(t)dt\right)\mathbf{j}$$

Example 6 Evaluate $\int_0^{\pi}((\cos t)\mathbf{i} + 2t\mathbf{j})dt$.

$\left[\sin t \Big|_0^{\pi}\right] i + \left[t^2\right]_0^{\pi} j \rightarrow (\sin\pi - \sin 0)i + (\pi^2 - 0^2)j \rightarrow \pi^2 j$

Example 7 Find an equation for $\mathbf{r}(t)$ and $\mathbf{v}(t)$ if $\mathbf{a}(t) = 2t\mathbf{i} - 5\mathbf{j}$ and $\mathbf{v}(0) = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$.

Example 8 The vector $\mathbf{r}(t) = (2t^3 - 3t^2)\mathbf{i} + (t^3 - 12t)\mathbf{j}$ gives the position of a particle at time t.

a) Write an equation for the line tangent to the path of the particle at the point where $t = -1$.

b) Find the coordinates of each point where the horizontal component of the velocity is zero.

c) Find the acceleration vector at time $t = 2$.