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2. For $t \geq 0$, a particle moving in the $xy$-plane has the position vector $\langle x(t), y(t) \rangle$ at time $t$, where $\frac{dx}{dt} = -1 + e^{\sin t}$ and $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the position of the particle is $(5, 7)$.

(a) Find the acceleration vector of the particle at time $t = 2$.

✓ acceleration $= \langle x''(t), y''(t) \rangle$ ← sufficient

$\frac{d^2x}{dt^2} = \cos t\, e^{\sin t}$

$\frac{d^2y}{dt^2} = -2t\sin(t^2)$

*since calculator active you don't have to show what $\frac{d^2x}{dt^2}$ or $\frac{d^2y}{dt^2}$ actually is.

$a(2) = \langle x''(2), y''(2) \rangle$ ✓

$= \langle -1.033, 3.027 \rangle$ ✓

(b) Find the total distance traveled by the particle over the time interval $1.8 \leq t \leq 2$.

Total distance $= \int_{1.8}^{2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\approx .360$

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(c) Find the $x$-coordinate of the position of the particle at time $t = 1$.

$$X(1) + \int_1^2 -1 + e^{\sin t}\,dt = 5$$

$$X(1) = 5 - \int_1^2 -1 + e^{\sin t}\,dt \approx 3.395$$

(d) At time $t = \sqrt{\frac{7\pi}{2}}$, the line tangent to the path of the particle is horizontal. Find the particle's speed at time $t = \sqrt{\frac{7\pi}{2}}$. Determine whether the particle is moving to the left or to the right at that time. Give a reason for your answer.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt} \to \cos(t^2)}{\frac{dx}{dt} \to -1 + e^{\sin t}} = 0$$

$$\left.\frac{dy}{dx}\right|_{t=\sqrt{\frac{7\pi}{2}}} = 0$$

unnecessary for this problem but ok if you write it.

← sufficient

$$\text{Speed} = \left.\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\right|_{t=\sqrt{\frac{7\pi}{2}}} \approx .159$$

$$\left.\frac{dx}{dt}\right|_{t=\sqrt{\frac{7\pi}{2}}} = -1 + e^{\sin\sqrt{\frac{7\pi}{2}}} \approx -.159$$

The particle is moving left since $x'\left(\sqrt{\frac{7\pi}{2}}\right) < 0$

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**NO CALCULATOR ALLOWED**

4. In a national park, the population of mountain lions grows over time. At time $t = 0$, where $t$ is measured in years, the population is found to be 20 mountain lions.

   (a) One zoologist suggests a population model $P$ that satisfies the differential equation $\frac{dP}{dt} = \frac{1}{4}(220 - P)$. Use separation of variables to solve this differential equation for $P$ with the initial condition $P(0) = 20$.

$$\int \frac{dP}{220-P} = \int \frac{1}{4}\,dt$$

$$-\ln|220-P| = \frac{1}{4}t + C$$

$$\ln|220-P| = -\frac{1}{4}t + C$$

$$220-P = Ce^{-\frac{1}{4}t}$$

$$220-20 = Ce^{-\frac{1}{4}(0)}$$

$$200 = C$$

$$220-P = 200e^{-\frac{1}{4}t}$$

$$-P = 200e^{-\frac{1}{4}t} - 220$$

$$P = -200e^{-\frac{1}{4}t} + 220$$

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Continue problem 4 on page 17.

**NO CALCULATOR ALLOWED**

(b) A second zoologist suggests a population model $Q$ that satisfies $\frac{dQ}{dt} = \frac{1}{500}Q(220 - Q)$. Find the value of $\frac{dQ}{dt}$ at the time when $Q$ grows most rapidly.

Q satisfies a logistic differential equation w/ carrying capacity 220. Q grows most rapidly when $Q = \frac{220}{2} = 110$.

$$\left.\frac{dQ}{dt}\right|_{Q=110} = \frac{1}{500}(110)(220-110) = \frac{110^2}{500} = \frac{12100}{500} = \frac{121}{5}$$

(c) For the population model $Q$ introduced in part (b), use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $Q(10)$. Show the computations that lead to your answer.

| old pt | dx | $m = \frac{dy}{dx}\left(\frac{dQ}{dt}\right)$ | $m\,dx = dy$ | new pt |
|---|---|---|---|---|
| (0, 20) | 5 | $\frac{1}{500}(20)(220-20) = 8$ | $8(5) = 40$ | (5, 60) |
| (5, 60) | 5 | $\frac{1}{500}(60)(220-60) = \frac{96}{5}$ | $\frac{96}{5}(5) = 96$ | (10, 156) |

$Q(10) \approx 156$

$$Y = \frac{L}{1+ce^{-LKt}} \rightarrow Q = \frac{220}{1+ce^{-220(\frac{1}{500})t}} \rightarrow Q = \frac{220}{1+ce^{-\frac{11}{25}t}} \rightarrow Q = \frac{220}{1+10e^{-11/25(t)}}$$

$$Q(0) = \frac{220}{1+ce^{-\frac{11}{25}(0)}} = 20 \rightarrow 220 = 20(1+c) \quad 11 = 1+c \rightarrow c = 10$$

Completely unnecessary! But if you needed to find "c" → do this.





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