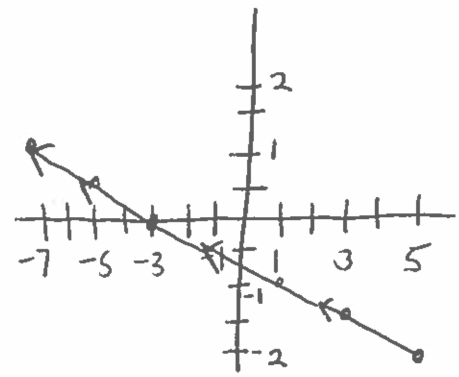
1.)

a.)

t	$1-2t$ x	$\frac{1}{2}t-1$ y
-2	5	-2
-1	3	-1.5
0	1	-1
1	-1	-0.5
2	-3	0
3	-5	0.5
4	-7	1



$x = 1 - 2t$

$x - 1 = -2t$

$\frac{x-1}{-2} = t$

$y = \frac{1}{2}\left(\frac{x-1}{-2}\right) - 1$

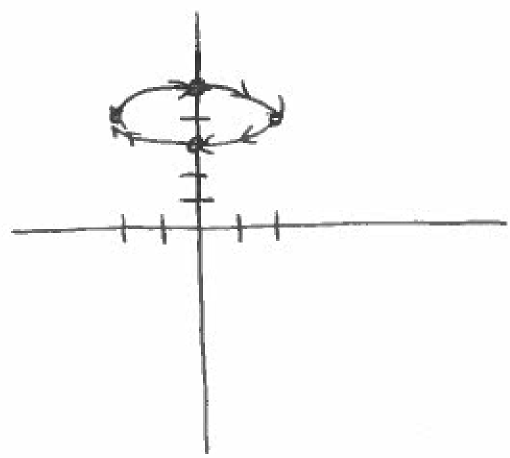
$y = \frac{x-1}{-4} - 1$

$y + 1 = \frac{-x}{4} + \frac{1}{4}$

$y = -\frac{1}{4}x - \frac{3}{4}$

b.)

t	$2\sin t$ x	$4+\cos t$ y
0	0	5
$\frac{\pi}{2}$	2	4
π	0	3
$3\pi/2$	-2	4
2π	0	5



$x = 2\sin t$

$\frac{x}{2} = \sin t$

$y = 4 + \cos t$

$y - 4 = \cos t$

$\cos^2 t + \sin^2 t = 1$

$(y-4)^2 + \left(\frac{x}{2}\right)^2 = 1$

$\frac{x^2}{4} + (y-4)^2 = 1$

2.) $x(t) = t^2 \qquad y(t) = t^3 - 3t$

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2 - 3$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-3}{2t} \rightarrow \frac{3}{2}t - \frac{3}{2}t^{-1}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}} = \frac{\frac{3}{2} + \frac{3}{2}t^{-2}}{2t} \rightarrow \frac{3}{4}t^{-1} + \frac{3}{4}t^{-3}$$

3.) $x(t) = t^3 - 3t \qquad y(t) = t^2 - 3$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2-3}$$

H.T when $2t = 0 \qquad t = 0$

V.T when $3t^2 - 3 = 0 \qquad t^2 = 1 \rightarrow t = \pm 1$

$x(0) = 0 \qquad y(0) = -3 \qquad$ H.T $(0, -3)$

$x(1) = -2 \qquad y(1) = -2 \qquad$ V.T $(-2, -2)$

$x(-1) = 2 \qquad y(-1) = -2 \qquad$ V.T $(2, -2)$

4.) $x(t) = \cos(2t) \qquad y(t) = \ln t$

$x'(t) = -2\sin(2t)$

$y'(t) = \frac{1}{t}$

$$L = \int_1^4 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt \approx 4.120$$

5.) $x(t) = t^4 - 2t + 3 \qquad y(t) = \sqrt{4t^2+9}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left.\frac{\frac{1}{2}(4t^2+9)^{-\frac{1}{2}}(8t)}{4t^3-2}\right|_{t=2} \approx 4/75$$

6.) $x(t) = \cos(3^t)$ $\quad y(t) = \sin(3^t)$

$x'(t) = -\sin(3^t) \cdot \ln 3 \cdot 3^t \quad y'(t) = \cos(3^t) \cdot \ln 3 \cdot 3^t$

↳ since calculator active, you wouldn't have to show this calculation?

$$\text{speed} = \left.\sqrt{(x'(t))^2 + (y'(t))^2}\right|_{t=2.4} \approx 15.343$$

7.) $x(2) = 1 \quad y(2) = 5$

$$\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t} \qquad \frac{dy}{dt} = \sin^2 t$$

a.) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{\sqrt{2+2}}{e^2} > 0$ Particle is moving right b/c $\left.\frac{dx}{dt}\right|_{t=2} > 0$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left.\frac{\sin^2 t}{\sqrt{t+2}/e^t}\right|_{t=2} \approx 3.055$$

b.) $1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\, dt \approx 1.253$

c.) $\text{speed} = \left.\sqrt{(x'(t))^2 + (y'(t))^2}\right|_{t=4} \approx .575$

$$a(4) = (x''(4), y''(4)) \approx (-.041, .989)$$

d.) $\text{Distance Traveled} = \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt \approx .651$

8.) $x(t) = t^2 - 4t + 8$ $\quad x'(t) = 2t - 4$

$\frac{dy}{dt} = te^{t-3} - 1$

a.) speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\Big|_{t=3} \approx 2.828$

b.) Total distance traveled $= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \approx 11.588$

c.) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ if $\frac{dy}{dt} = 0 \rightarrow$ calculator $\rightarrow t = 2.208$

$x'(2.208) = 2(2.208) - 4 \approx .416 > 0 \rightarrow$ particle is moving right at $t = 2.208$ since $x'(2.208) > 0$.

d.) (i) $x(t) = t^2 - 4t + 8$

$5 = t^2 - 4t + 8$

$0 = t^2 - 4t + 3$

$0 = (t-3)(t-1) \quad t = 1, 3$

(ii) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \frac{dy}{dx}\Big|_1 \approx .432 \quad \frac{dy}{dx}\Big|_3 \approx 1$

(iii) $3 + \frac{1}{e} + \int_2^3 (te^{t-3} - 1)\, dt \approx 4$

20) $\frac{dP}{dt} = kP(1000-P)$ $\quad$ * $y = \frac{L}{1+ce^{-Lkt}}$

so $P = \frac{1000}{1+ce^{-1000kt}}$

$100 = \frac{1000}{1+ce^{-1000k(0)}}$ (2yrs ago $\rightarrow t=0$)

$100 = \frac{1000}{1+c} \rightarrow 100 + 100c = 1000 \quad c = 9$

$400 = \frac{1000}{1+9e^{-1000k(1)}}$ (1yr ago $t=1$)

$400 + 3600e^{-1000k} = 1000$

$3600e^{-1000k} = 600$

$e^{-1000k} = \frac{1}{6}$

$-1000k = \ln(1/6)$

$k = \frac{\ln 1 - \ln 6}{-1000} \rightarrow k = \frac{\ln 6}{1000}$

now $\rightarrow t=2$

$P = \frac{1000}{1+9e^{-1000\left(\frac{\ln 6}{1000}\right)(2)}} = 800$ squirrels

21.) $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right)$

*Remember

$\frac{dy}{dt} = ky(L-y)$

$\frac{dP}{dt} = \frac{1}{60} P(12-P)$ (multiply inside by 12, divide outside by 12)

a.) $\lim_{t \to \infty} P(t)$ is carrying capacity $= 12$

b.) P is growing fastest when $P = \frac{L}{2} = \frac{12}{2} = 6$

c.) $\frac{d^2P}{dt^2} = \frac{1}{5}\frac{dP}{dt} - \frac{2}{60} P \frac{dP}{dt} > 0$ if $P < 6$

the rate of of population growth is increasing since $\frac{d^2P}{dt^2} > 0$

* the initial condition makes no sense! Ignore.

* you don't actually need to find $\frac{d^2P}{dt^2}$ on this problem → but you may in the future on another question asking something else.

ⓐ

1.) $\text{Area} = \int_0^1 2x^3\,dx + \int_1^3 3-x\,dx$

or $\int_0^2 \left((3-y) - \sqrt[3]{\frac{y}{2}}\right) dy \quad = 2.5$

ⓑ $V = \int_0^2 \pi \frac{\left(3-y-\sqrt[3]{y/2}\right)^2}{8}\,dy \approx 1.687$

$*$Area circle $= \pi r^2 \rightarrow \pi\left(\frac{d}{2}\right)^2 \rightarrow \frac{\pi d^2}{4}$ semicircle $\rightarrow \frac{\frac{\pi d^2}{4}}{2} = \frac{\pi d^2}{8}$

ⓐ

2.) $V = \pi\int_0^4 \left(\left(\frac{1}{x+2}+1\right)^2 - (0+1)^2\right)dx \approx 7.950$

ⓑ $V = \pi\int_0^4 \left((3-0)^2 - \left(3-\frac{1}{x+2}\right)^2\right)dx \approx 19.661$

ⓒ $\int_0^k \frac{1}{x+2}\,dx = \int_k^4 \frac{1}{x+2}\,dx$

$\ln|x+2|\Big|_0^k = \ln|x+2|\Big|_k^4$

$\ln|k+2| - \ln 2 = \ln 6 - \ln|k+2|$

$2\ln|k+2| = \ln 6 + \ln 2$

$\ln|k+2| = \frac{\ln 12}{2}$

$\ln|k+2| = \ln 12^{\frac{1}{2}}$

$k+2 = 12^{\frac{1}{2}}$

$k = \sqrt{12} - 2$

≈ 1.464

Area

2.) d $a \triangle a$, a $\rightarrow \frac{1}{2}a \cdot a \sin 60 = \frac{1}{2}a^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$

$$V = \int_0^4 \frac{\sqrt{3}}{4}\left(\frac{1}{x+2} - 0\right)^2 dx \approx .144$$

3.) $L = \int_0^{\pi} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$; $\frac{dy}{dx} = -x\sin x + \cos x$

≈ 5.519

4.) $L = \int_{-1}^{3} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$; $\frac{dx}{dy} = y + 1$

≈ 9.294

5.) a $a(t) = v'(t) \rightarrow a(4) = v'(4) \approx .714$

b $v(t) = 0 \rightarrow t = 1$, $t = 2$ b/c $v(t)$ changes signs, particle changes direction

$v(t) < 0$ between $t = 1$ & $t = 2$ so the particle travels left when $1 < t < 2$

(c) $8 + \int_0^2 v(t)\,dt = 8.369$

(d) Avg speed $= \dfrac{\int_0^2 |v(t)|\,dt}{2-0}$

$\approx .371$