1) If $x = t^3 + 6t$ and $y = 2e^{4t}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \frac{8e^{4t}}{3t^2+6}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(3t^2+6)(32e^{4t}) - 8e^{4t}(6t)}{(3t^2+6)^2}}{(3t^2+6)} = \frac{(3t^2+6)(32e^{4t}) - 48te^{4t}}{(3t^2+6)^3}$$

2) Find the arc length of the curve defined by $x = e^{2t} - \frac{t}{8}$ and $y = e^t$ over the interval $0 \le t \le \ln 2$

$$L = \int_0^{\ln 2} \sqrt{\left(2e^{2t} - \tfrac{1}{8}\right)^2 + (e^t)^2}\, dt \approx \boxed{3.087}$$

3. If $u = \langle 5,2\rangle$ $v = \langle -3,4\rangle$ Find

a) $5u + 2v \rightarrow \langle 25,10\rangle + \langle -6,8\rangle = \langle 19,18\rangle$

b) $|2u+v| \rightarrow \langle 10,4\rangle + \langle -3,4\rangle = \langle 7,8\rangle$ $\sqrt{49+64} = \sqrt{113}$

c) $u \cdot v$ $5(-3)+2(4) = -15+8 = -7$

d) unit vector in the direction of u. $\sqrt{5^2+2^2} = \sqrt{29}$ $\left\langle \frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}} \right\rangle$

4. Given $a(t) = 2j$, find $v(t)$ and $\underline{r(t)}$ if $v(0) = 0i + 0j$ and $r(0) = i$

pos.

$a(t) = 0i + 2j$

$v(t) = 2tj + C$

$0i + 0j = 2(0)j + C$

$0 = C$

$\boxed{v(t) = 2tj}$ $\langle 0, 2t\rangle$

$v(t) = 0i + 2tj$

$r(t) = t^2 j + c$

$i = 0^2 j + c$ $c = i$

$\boxed{r(t) = i + t^2 j}$ $\langle 1, t^2\rangle$

5. Find the unit tangent and normal vectors for the curve defined by $x = t$ and $y = t^2$ at time $t = 3$.

$x' = 1$ $\quad y' = 2t \quad y'(3) = 6 \quad \langle 1, 6 \rangle$

$\sqrt{1^2 + 6^2} = \sqrt{37}$

unit tan

$\left\langle \frac{1}{\sqrt{37}}, \frac{6}{\sqrt{37}} \right\rangle \left\langle \frac{-1}{\sqrt{37}}, \frac{-6}{\sqrt{37}} \right\rangle$

Unit Norm

$\left\langle \frac{-6}{\sqrt{37}}, \frac{1}{\sqrt{37}} \right\rangle \left\langle \frac{6}{\sqrt{37}}, \frac{-1}{\sqrt{37}} \right\rangle$

6. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dx}$ is not explicitly given. At time $t = 2$, the object is at position (1, 8).

$x(4) = 1 + \int_2^4 (3 + \cos(t^2))\,dt \approx \boxed{7.133}$ a

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt} = -7$. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$. $m = -2.983$ $(1, 8)$ $\boxed{y - 8 = -2.983(x - 1)}$ b

(c) Find the speed of the object at time $t = 2$. $\sqrt{(2.346)^2 + (-7)^2} \approx \boxed{7.383}$ c

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$. ON NEXT SLIDE

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dx}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t+1$. Find the acceleration vector of the object at time $t = 4$.

$$\frac{dy}{dx} = 2t+1$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{\frac{dy}{dt}}{3+\cos(t^2)} = 2t+1$$

$\frac{dx}{dt} \rightarrow 3+\cos(t^2)$

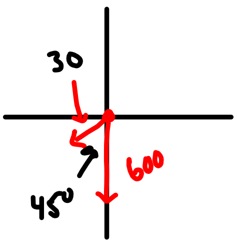
$$\frac{dy}{dt} = (2t+1)(3+\cos(t^2))$$

take derivative of dx/dt and plug-in 4

take derivative of $\frac{dy}{dt}$ and plug-in 4.

$$a(4) = \langle 2.303, 24.814 \rangle$$

7. An airplane is headed due South at a speed of 600 mph and encounters a 30 mph wind blowing in the southwest direction (45° S of W). The airplane maintains its compass heading but, because of the wind, acquires a new ground speed and direction. Find the new ground speed in mph and the new direction in degrees of the plane. Make sure to change your calculator to DEGREE mode and change it back to radian mode when you are finished.



plane $= \langle 0, -600 \rangle$

Wind $= \langle 30\cos 225°, 30\sin 225° \rangle = \langle -15\sqrt{2}, -15\sqrt{2} \rangle$

p+w $= \langle -15\sqrt{2}, -600-15\sqrt{2} \rangle$

$$\boxed{|p+w| = 621.575 \text{ mph}}$$

$$\theta \approx \cos^{-1}\left(\frac{\cancel{0\cdot}\ \cancel{-600}(-600-15\sqrt{2})}{\cancel{600}\cdot 621.575}\right)$$

$$\boxed{\theta \approx 1.956° \text{ W of S}}$$