BC Calculus Integration by Parts Notesheet **Name:** ___________________________________

When you integrate, you find the antiderivative, or you "undo" the derivative. When you used u-substitution, you were undoing a derivative that involved the Chain Rule. Today, we will learn how to undo a derivative that involves the product rule.

First, recall the way we differentiate functions using the Product Rule and differential notation.

$$(uv)' = udv + vdu$$

Using the Product Rule, we can develop a formula for integration by parts

$\int (uv)' = \int uv' + \int vu'$

$uv = \int uv' + \int vu'$

$\boxed{\int uv' = uv - \int vu'}$

Integration by Parts

If u and v are functions of x and have continuous derivatives, then

$$\boxed{\int u\,dv = uv - \int v\,du}$$

"Oohvee vadoo"

The trick with integration by parts is determining which function to select as your "u." When selecting u, think of the word LIPET → helpful in determing what to set = to "u"

Logarithmic **I**nverse Trig **P**olynomial (algebra) **E**xponential **T**rigonometric

Example 1 Evaluate $\int x \cos x\, dx$

$u = x \quad \int dv = \int \cos x\, dx$

$\frac{du}{dx} = 1 \quad v = \sin x$

$du = dx$

$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = \boxed{x\sin x + \cos x + C}$

Example 2 Evaluate $\int 2xe^{4x}\,dx$

$u = 2x \quad \int dv = \int e^{4x}\,dx$

$du = 2dx \quad v = \frac{1}{4}e^{4x}$

$\int 2xe^{4x}dx = 2x\,\frac{1}{4}e^{4x} - \int \frac{1}{4}e^{4x}\,2dx$

$= \frac{1}{2}xe^{4x} - \frac{1}{2}\int e^{4x}dx$

$= \frac{1}{2}xe^{4x} - \frac{1}{2}\cdot\frac{1}{4}e^{4x} + c$

$\boxed{= \frac{1}{2}xe^{4x} - \frac{1}{8}e^{4x} + c}$

Example 3 Evaluate $\int x \ln x\, dx$

$u = \ln x \quad \int dv = \int x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$\int x\ln x\,dx = \ln x\,\frac{x^2}{2} - \int \frac{x^2}{2}\,\frac{1}{x}\,dx$ (simplify)

$\frac{x^2}{2}\ln x - \int \frac{1}{2}x\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\,\frac{x^2}{2} + c$

$\boxed{= \frac{x^2}{2}\ln x - \frac{x^2}{4} + c}$

Example 4 Evaluate $\int x \sec^2 x\, dx$

$u = x \quad \int dv = \int \sec^2 x\,dx$

$du = dx \quad v = \tan x$

$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$

$\boxed{= x\tan x + \ln|\cos x| + c}$

Sometimes, an integration by parts problem will try to disguise itself.

Example 5 Evaluate $\int \ln x\, dx$

$u = \ln x \quad \int dv = \int dx$

$du = \frac{1}{x}dx \quad v = x$

$\int \ln x\,dx = \ln x\; x - \int x\,\frac{1}{x}\,dx$

$= x\ln x - \int 1\,dx = \boxed{x\ln x - x + c}$

Example 6 Evaluate $\int \arctan x\, dx$

$u = \tan^{-1}x \quad \int dv = \int dx$

$du = \frac{1}{1+x^2}dx \quad v = x$

$\int \arctan x = x\tan^{-1}x - \int x\,\frac{1}{1+x^2}dx$

$u = 1 + x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int \frac{1}{u}\,du$

$= -\frac{1}{2}\ln|u| + c$

$\boxed{= x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + c}$