Sometimes, we have to repeat ourselves. Sometimes, we have to repeat ourselves.

Example 7 Evaluate $\int x^2 \sin x \, dx$

$u = x^2$ $\quad \int dv = \int \sin x \, dx$

$du = 2x\,dx$ $\quad v = -\cos x$

$-x^2 \cos x - \int -\cos x \; 2x \, dx$

$-x^2 \cos x + 2\int x \cos x \, dx$

$u = x$ $\quad \int dv = \int \cos x \, dx$

$du = dx$ $\quad v = \sin x$

$x \sin x - \int \sin x \, dx$

$x \sin x + \cos x$

$-x^2 \cos x + 2x\sin x + 2\cos x + C$

We have seen that integrals of the form $\int f(x)g(x)dx$, in which f can be differentiated repeatedly to become zero and g can be integrated repeatedly without difficulty, are natural candidates for integration by parts. However, if many repetitions are required, the calculations can become cumbersome. In situations like this, using a table can help organize the calculations and speed the process up.

Use table shortcut IF the derivative of u eventually equals 0.

Example 8 Evaluate $\int x^2 \sin x \, dx$ using the tabular method

$u = x^2 \rightarrow$ shortcut good

u		dv
x^2	+	$\sin x$
$2x$	−	$-\cos x$
2	+	$-\sin x$
0		$\cos x$

derive ↓ integrate ↓

$-x^2 \cos x + 2x\sin x + 2\cos x + C$

Example 9 Evaluate $\int t^4 e^{2t} \, dt$

u dv

u		dv
t^4	+	e^{2t}
$4t^3$	−	$\frac{1}{2}e^{2t}$
$12t^2$	+	$\frac{1}{4}e^{2t}$
$24t$	−	$\frac{1}{8}e^{2t}$
24	+	$\frac{1}{16}e^{2t}$
0		$\frac{1}{32}e^{2t}$

derive ↓ integrate ↓

$$\frac{1}{2}t^4 e^{2t} - t^3 e^{2t} + \frac{12}{8}t^2 e^{2t} - \frac{24}{16}t e^{2t} + \frac{24}{32}e^{2t} + C$$

Example 10 Evaluate $\int x^3 \cos(2x) \, dx$

u		dv
x^3	+	$\cos(2x)$
$3x^2$	−	$\frac{1}{2}\sin(2x)$
$6x$	+	$-\frac{1}{4}\cos(2x)$
6	−	$-\frac{1}{8}\sin(2x)$
0		$\frac{1}{16}\cos(2x)$

$$\frac{1}{2}x^3\sin(2x) + \frac{3}{4}x^2\cos(2x) - \frac{3}{4}x\sin(2x) - \frac{3}{8}\cos(2x) + C$$

What if neither function goes to zero?

Example 11 Evaluate $\int e^x \cos(x) \, dx$

$u = e^x$ $\quad dv = \cos x \, dx$

$du = e^x dx$ $\quad v = \sin x$

$e^x \sin x - \int \sin x \cdot e^x \, dx$

$u = e^x$ $\quad dv = \sin x \, dx$

$du = e^x dx$ $\quad v = -\cos x$

$-e^x \cos x - \int -\cos x \, e^x \, dx$

$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x dx$

*add like terms to both sides

$\frac{2}{2}\int e^x \cos x \, dx = \frac{e^x \sin x + e^x \cos x}{2}$

$\int e^x \cos x \, dx = \frac{e^x \sin x + e^x \cos x}{2} + C$