AP Calculus Polar Curves and Derivatives Homework B **Name:** ______________________________

1. Find $dr/d\theta$ of the curve $r=-1+\sin\theta$ at $\theta=0$ and $\theta=\pi$.

$\frac{dr}{d\theta}=\cos\theta \qquad \left.\frac{dr}{d\theta}\right|_{\theta=0}=\cos 0=1 \qquad \left.\frac{dr}{d\theta}\right|_{\theta=\pi}=\cos\pi=-1$

2. Find the slope of the curve $r=-1+\sin\theta$ at $\theta=0$ and $\theta=-\frac{\pi}{2}$.

$y=r\sin\theta$

$y=(-1+\sin\theta)(\sin\theta)$

$y=-\sin\theta+\sin^2\theta$

$\frac{dy}{d\theta}=-\cos\theta+2\sin\theta\cos\theta$

$\boxed{\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}}$

$\left.\frac{dy}{dx}\right|_{\theta=0}=\frac{-1+2(0)(1)}{0-(0)^2+(1)^2}=-1$

$\left.\frac{dy}{dx}\right|_{\theta=-\frac{\pi}{2}}=\frac{0+2(-1)(0)}{-1-(-1)^2+0}=\frac{0}{-2}=0$

$x=r\cos\theta$

$x=(-1+\sin\theta)\cos\theta$

$x=-\cos\theta+\sin\theta\cos\theta$

$\frac{dx}{d\theta}=\sin\theta+\sin\theta(-\sin\theta)+\cos\theta\cos\theta$

$\frac{dx}{d\theta}=\sin\theta-\sin^2\theta+\cos^2\theta$

3. Find the equation of the tangent line to the curve $r=3\cos\theta$ when $r=0$ and $0\le\theta\le 2\pi$

$(x,y)\quad \frac{dy}{dx}$

$y=r\sin\theta=3\cos\theta\sin\theta \rightarrow \frac{dy}{d\theta}=3(\cos\theta\cos\theta+(-\sin\theta)(\sin\theta))=3(\cos^2\theta-\sin^2\theta)$

$x=r\cos\theta=3\cos\theta\cos\theta=3\cos^2\theta \rightarrow \frac{dx}{d\theta}=6\cos\theta(-\sin\theta)=-6\cos\theta\sin\theta$

$0=3\cos\theta$

$0=\cos\theta$

$\theta=\frac{\pi}{2},\frac{3\pi}{2}$

$x(\frac{\pi}{2})=3(0)^2=0 \quad y(\frac{\pi}{2})=3(0)(1)=0$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}}=\frac{3(0^2-1^2)}{-6\ 0\ 1}=\frac{-3}{0}\rightarrow$ und Vertical Line $\boxed{x=0}$

$x(\frac{3\pi}{2})=0$

$y(\frac{3\pi}{2})=0$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}}=\frac{-3}{0}\rightarrow$ und

Vertical Line at $\boxed{x=0}$

4. Find values of θ where the tangent lines to the curve $r=2\sin\theta$ are horizontal and vertical over $0\le\theta\le\pi$.

Hor if $\frac{dy}{d\theta}=0$ (and $\frac{dx}{d\theta}\neq 0$)

$y=r\sin\theta$

$y=2\sin\theta\sin\theta$

$y=2\sin^2\theta$

$\frac{dy}{d\theta}=4\sin\theta\cos\theta=0$

If $\sin\theta=0\rightarrow \boxed{\theta=0,\pi}$

If $\cos\theta=0\rightarrow \boxed{\theta=\pi/2}$

Vertical tangent $\frac{dx}{d\theta}=0$ (and $\frac{dy}{d\theta}\neq 0$)

$x=r\cos\theta$

$x=2\sin\theta\cos\theta$

$\frac{dx}{d\theta}=2(\sin\theta(-\sin\theta)+\cos\theta\cos\theta)=2(\cos^2\theta-\sin^2\theta)$

$2(\cos^2\theta-\sin^2\theta)=0$

If $\cos^2\theta=\sin^2\theta$ that happens at $\boxed{\theta=\frac{\pi}{4},\frac{3\pi}{4}}$

5. Find two equivalent polar coordinates and the Rectangular coordinates for the polar coordinate pair.

$\left(-3,\ \frac{2\pi}{3}\right)$

$\left(-3,\frac{8\pi}{3}\right) \quad \left(-3,-\frac{4\pi}{3}\right)$

$\left(3,-\frac{\pi}{3}\right) \quad \left(3,\frac{5\pi}{3}\right)$

$x=r\cos\theta$

$x=-3\cos\left(\frac{2\pi}{3}\right)$

$=-3\left(-\frac{1}{2}\right)$

$=\frac{3}{2}$

$y=r\sin\theta$

$y=-3\sin\left(\frac{2\pi}{3}\right)$

$=-3\left(\frac{\sqrt{3}}{2}\right)$

$=\frac{-3\sqrt{3}}{2}$

$\left(\frac{3}{2},\frac{-3\sqrt{3}}{2}\right)$

6. Find two equivalent pairs of polar coordinates for the rectangular coordinate pair.

(x,y)

$(-1,1)$

$x^2+y^2=r^2$

$(-1)^2+(1)^2=r^2$

$1+1=r^2$

$2=r^2\rightarrow r=\pm\sqrt{2}$

$\tan\theta=\frac{1}{-1}$

$\tan\theta=-1$

$\theta=\frac{3\pi}{4}$ or $\frac{7\pi}{4}$

$\left(\sqrt{2},\frac{3\pi}{4}\right) \quad \left(\sqrt{2},-\frac{5\pi}{4}\right)$

$\left(-\sqrt{2},\frac{7\pi}{4}\right) \quad \left(\sqrt{2},-\frac{\pi}{4}\right)$

7. Replace the polar equation with an equivalent rectangular equation.

a) $r = \dfrac{5}{sin\theta - 2cos\theta}$

$r(\sin\theta - 2\cos\theta) = 5$

$r\sin\theta - 2r\cos\theta = 5$

$\boxed{y - 2x = 5}$

or $\boxed{y = 2x + 5}$

b) Replace the rectangular equation with an equivalent polar equation. Your equation should be in $r =$ form.

$y^2 + (x-3)^2 = 9$

$y^2 + x^2 - 6x + 9 = 9$

$\dfrac{r^2}{r} - \dfrac{6r\cos\theta}{r} = \dfrac{0}{r}$ $\boxed{r = 6\cos\theta}$

$r - 6\cos\theta = 0$

8. Find an equation of the tangent to the curve $x = 1 + \ln t$ and $y = t^2 + t$ at the point $(1, 2)$.

$x, y, \frac{dy}{dx}$ $\frac{dx}{dt} = \frac{1}{t}$ $\frac{dy}{dt} = 2t + 1$

$1 + \ln t = 1 \rightarrow t = 1$ b/c $\ln 1 = 0$

$t^2 + t = 2 \rightarrow t = 1$ b/c $1^2 + 1 = 2$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $\frac{dy}{dx} = \frac{2t+1}{1/t} = 2t^2 + t$

$\left.\frac{dy}{dx}\right|_{t=1} = 2(1)^2 + 1 = 3$ $\boxed{y - 2 = 3(x - 1)}$

9. Find $\frac{dy}{dx}$ and d^2y/dx^2 of the curve defined by the parametric functions $x = t^3 + 1$ and $y = t^2 - t$.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t-1}{3t^2} \rightarrow$ Since I'm taking the derivative again, I'll split apart to avoid quotient rule $\rightarrow \frac{2}{3}t^{-1} - \frac{1}{3}t^{-2}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{-\frac{2}{3}t^{-2} + \frac{2}{3}t^{-3}}{3t^2} \rightarrow$ If mult choice $\rightarrow -\frac{2}{9}t^{-4} + \frac{2}{9}t^{-5}$

10. Find the length of the curve defined by the functions $x = 1 + 3t^2$ and $y = 4 + 2t^3$ over $0 \le t \le 2$.

$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ $\frac{dx}{dt} = 6t$ $\frac{dy}{dt} = 6t^2$

$\int_0^2 \sqrt{(6t)^2 + (6t^2)^2}\, dt \overset{MC}{=} \int_0^2 \sqrt{36t^2 + 36t^4}\, dt \approx \boxed{20.361}$