BCCALC Polar Review Sheet Solutions

1. Convert the following points to rectangular form.

a) $\left(-4, -\frac{\pi}{3}\right)$

$\left(-4\cos\left(-\frac{\pi}{3}\right), -4\sin\left(-\frac{\pi}{3}\right)\right)$

$\left(-4\left(\frac{1}{2}\right), -4\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\boxed{(-2, 2\sqrt{3})}$

b) $(-8, \pi)$ to polar

$(-8\cos(\pi), -8\sin\pi)$

$\boxed{(8, 0)}$

2. Convert the following points to polar form.

a) $(4, 4)$ 1st Quad

$r = \sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}$

$\tan\theta = \frac{4}{4} = 1$

$\theta = \frac{\pi}{4}$

$\boxed{\left(4\sqrt{2}, \frac{\pi}{4}\right)}$

b) $(0, \sqrt{6})$

$r = \sqrt{0^2+(\sqrt{6})^2} = \sqrt{6}$

$\tan\theta = \frac{\sqrt{6}}{0} = \text{undefined}$

$\theta = \frac{\pi}{2}$ *Not $\frac{3\pi}{2}$ since y is positive

$\boxed{\left(\sqrt{6}, \frac{\pi}{2}\right)}$

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3. Convert the following equations to polar form.

a) $y = 4$

$r\sin\theta = 4$

$r = \frac{4}{\sin\theta}$

$r = 4\csc\theta$

b) $3x - 5y + 2 = 0$

$3r\cos\theta - 5r\sin\theta + 2 = 0$

$3r\cos\theta - 5r\sin\theta = -2$

$r(3\cos\theta - 5\sin\theta) = -2$

$r = \frac{-2}{3\cos\theta - 5\sin\theta}$

c) $x^2 + y^2 = 25$

$r^2 = 25$

$r = 5$

4. Convert the following equations to rectangular form.

a) $r = 3\sec\theta$

$r = \frac{3}{\cos\theta}$

$r\cos\theta = \frac{3\cos\theta}{\cos\theta}$

$x = 3$

b) $r = 2\sin\theta$

$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2y$

$x^2 + y^2 - 2y = 0 \quad \left(\frac{(-2)}{2}\right)^2 = 1$

$x^2 + y^2 - 2y + 1 = 1$

$x^2 + (y-1)^2 = 1$

c) $\theta = \frac{5\pi}{6}$

$\tan\theta = \tan\frac{5\pi}{6}$

$\frac{y}{x} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$-\frac{\sqrt{3}}{2}y = \frac{1}{2}x$

$y = \frac{1}{2}\cdot\frac{2}{\sqrt{3}}x$

$y = -\frac{1}{\sqrt{3}}x$

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5. Find the slope of the polar curve at the indicated point.

a) $r = 1 - \sin\theta$ at $\theta = 0$ b) $r = 6(1 + \cos\theta)$ at $\theta = \frac{\pi}{2}$ c) $r = 5\cos(3\theta)$ at $\theta = \frac{\pi}{3}$

$r\cos\theta = \cos\theta - \sin\theta\cos\theta$

$x = \cos\theta - \sin\theta\cos\theta$

$x' = -\sin\theta - \sin\theta(-\sin\theta) + \cos\theta(-\cos\theta)$

$x'(0) = 0 + 0 + 1(-1)$

$x'(0) = -1$

$r\sin\theta = \sin\theta - \sin^2\theta$

$y = \sin\theta - \sin^2\theta$

$y' = \cos\theta - 2\sin\theta\cos\theta$

$y'(0) = 1 - 0 = 1$

$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{1}{-1} = \boxed{-1}$

5. Find the slope of the polar curve at the indicated point.

a) $r = 1 - \sin\theta$ at $\theta = 0$ b) $r = 6(1 + \cos\theta)$ at $\theta = \frac{\pi}{2}$ c) $r = 5\cos(3\theta)$ at $\theta = \frac{\pi}{3}$

$r\cos\theta = 6\cos\theta(1+\cos\theta)$

$x = 6\cos\theta(1+\cos\theta)$

$x' = 6\cos\theta(-\sin\theta) + (1+\cos\theta)(-6\sin\theta)$

$x'\left(\frac{\pi}{2}\right) = 0 + (1+0)(-6) = -6$

$r\sin\theta = 6\sin\theta(1+\cos\theta)$

$y = 6\sin\theta(1+\cos\theta)$

$y' = 6\sin\theta(-\sin\theta) + (1+\cos\theta)6\cos\theta$

$y'\left(\frac{\pi}{2}\right) = 6(1)(-1) + (1+0)(0) = -6$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{-6}{-6} = \boxed{1}$

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5. Find the slope of the polar curve at the indicated point.

a) $r = 1 - \sin\theta$ at $\theta = 0$ b) $r = 6(1 + \cos\theta)$ at $\theta = \frac{\pi}{2}$ c) $r = 5\cos(3\theta)$ at $\theta = \frac{\pi}{3}$

$r\cos\theta = 5\cos\theta\cos(3\theta)$

$x = 5\cos\theta\cos(3\theta)$

$x' = 5\cos\theta\cdot(-\sin(3\theta)\cdot 3) + \cos(3\theta)(-5\sin\theta)$

$x'\left(\frac{\pi}{3}\right) = 5\left(\frac{1}{2}\right)(0) + (-1)\left(-5\left(\frac{\sqrt{3}}{2}\right)\right)$

$x'\left(\frac{\pi}{3}\right) = \frac{5\sqrt{3}}{2}$

$r\sin\theta = 5\cos(3\theta)\sin\theta$

$y = 5\cos(3\theta)\sin\theta$

$y' = 5\cos(3\theta)\cos\theta + \sin\theta(-15\sin(3\theta))$

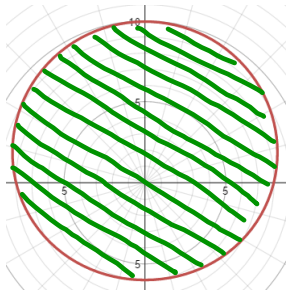
$y'\left(\frac{\pi}{3}\right) = 5(-1)\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}(-15(0))$

$y'\left(\frac{\pi}{3}\right) = \frac{-5}{2}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{\frac{-5}{2}}{\frac{5\sqrt{3}}{2}} \cdot \frac{2}{5\sqrt{3}} = \boxed{\frac{-1}{3}}$

6. Find the area of the following regions.

a) inside $r = 8 + 2\sin\theta$



$A = \frac{1}{2}\int_0^{2\pi} (8 + 2\sin\theta)^2 d\theta$

$\boxed{A \approx 207.345}$

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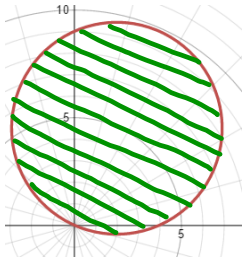
6. Find the area of the following regions.

b) Inside $r = 4\cos\theta + 9\sin\theta$

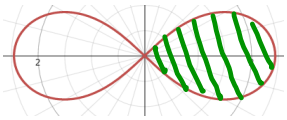


$$A = \frac{1}{2}\int_0^{\pi} (4\cos\theta + 9\sin\theta)^2 \, d\theta$$

$$A \approx 76.184$$

6. Find the area of the following regions.

c) Inside one loop of $r^2 = 6\cos(2\theta)$



$$A = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (6\cos 2\theta) \, d\theta$$

$$A \approx 3$$

$0 = 6\cos 2\theta$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2} \quad 2\theta = -\frac{\pi}{2}$

$\theta = \frac{\pi}{4} \quad \theta = -\frac{\pi}{4}$

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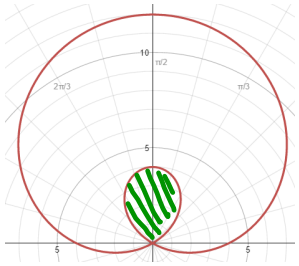
6. Find the area of the following regions.

d) Inner loop of $r = 4 + 8\sin\theta$



$$A = \frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (4+8\sin\theta)^2 \, d\theta$$

$$A \approx 8.696$$

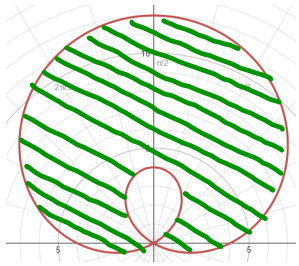
$4+8\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

6. Find the area of the following regions.

e) Between the loops in $r = 4 + 8\sin\theta$



WHOLE − 2 LOOPS

$$A = \frac{1}{2}\int_{0}^{2\pi} (4+8\sin\theta)^2 \, d\theta - 2\cdot\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (4+8\sin\theta)^2 \, d\theta$$

$$A \approx 133.404$$

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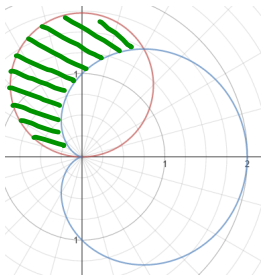
6. Find the area of the following regions.

f) Inside $r = \sqrt{3}\sin\theta$ and outside $r = 1 + \cos\theta$



$\sqrt{3}\sin\theta = 1+\cos\theta$

$\sqrt{3}\sin\theta - \cos\theta = 1$

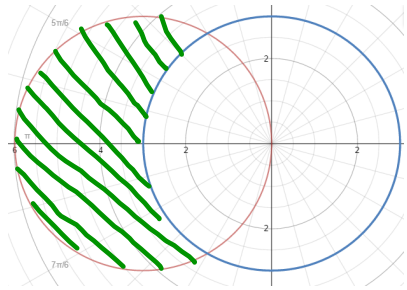
In Calc (Function mode)

$x \approx 1.047 \to \frac{\pi}{3}$ $x \approx 3.142 \to \pi$

$$A = \frac{1}{2}\int_{\frac{\pi}{3}}^{\pi}\left[(\sqrt{3}\sin\theta)^2 - (1+\cos\theta)^2\right]d\theta \approx \boxed{1.299}$$

6. Find the area of the following regions.

g) Inside $r = -6\cos\theta$ and outside $r = 3$



$-6\cos\theta = 3$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$A = \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}\left[(-6\cos\theta)^2 - (3)^2\right]d\theta \approx 17.219$$

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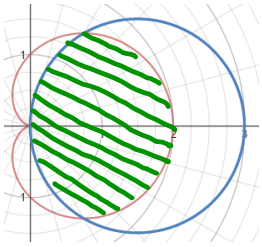
6. Find the area of the following regions.

h) Common interior of $r = 1 + \cos\theta$ and $r = 3\cos\theta$



$1+\cos\theta = 3\cos\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = -\frac{\pi}{3}$ $\theta = \frac{\pi}{3}$

$$A = \frac{1}{2}\int_0^{\pi} (3\cos\theta)^2 d\theta - \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left[(3\cos\theta)^2 - (1+\cos\theta)^2\right] d\theta \approx \boxed{3.927}$$

7. Find the length of each polar curve.

a) $r = 2\sin\theta + 2\cos\theta, \quad 0 \le \theta \le \frac{\pi}{2}$

$r' = 2\cos\theta - 2\sin\theta$

$$L = \int_0^{\frac{\pi}{2}} \sqrt{(2\sin\theta+2\cos\theta)^2 + (2\cos\theta-2\sin\theta)^2}\, d\theta$$

$\boxed{L \approx 4.443}$

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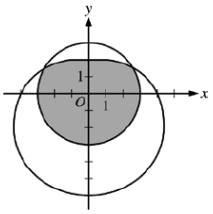
7. Find the length of each polar curve.

b) $r = \sqrt{1+\cos(2\theta)}, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$r' = \frac{1}{2}(1+\cos(2\theta))^{-\frac{1}{2}}(-\sin(2\theta)\cdot 2)$

$L = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left(\sqrt{1+\cos(2\theta)}\right)^2 + \left(\frac{-\sin 2\theta}{\sqrt{1+\cos 2\theta}}\right)^2} \approx 4.443$

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

(a) Area $= 6\pi + \frac{1}{2}\int_{\pi/6}^{5\pi/6}(4 - 2\sin\theta)^2\, d\theta =$ 24.709 (or 24.708)

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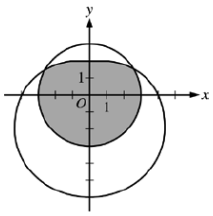


(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

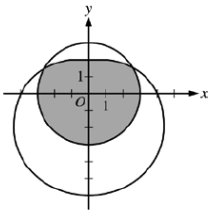
(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

(b) $x = r\cos\theta \Rightarrow x(\theta) = (4 - 2\sin\theta)\cos\theta$

$x(t) = \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right)$

$x(t) = -1$ when $t = 1.428$ (or 1.427)

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

(c) $y = r\sin\theta \Rightarrow y(\theta) = (4 - 2\sin\theta)\sin\theta$

$y(t) = \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right)$

Position vector $= \langle x(t), y(t)\rangle$

$= \left\langle \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right), \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right)\right\rangle$

$v(1.5) = \langle x'(1.5), y'(1.5)\rangle$

$= \langle -8.072, -1.673\rangle$ (or $\langle -8.072, -1.672\rangle$)

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