BC Calculus Power Series Practice Name: ___________________________

1. What is the radius of convergence of a power series? How do you find it?

the distance (x-values) from the center to the edge of convergence

2. What is the interval of convergence of a power series? How do you find it?

all x-values that allow convergence

do the ratio test

3. Find the radius of convergence and interval of convergence for each of the following series.

a) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)^{n+1}}\cdot\frac{n^n}{(x-2)^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{x-2}{(n+1)^n}\cdot\frac{n^n}{(n+1)}\right|$$

EBM = 1

$$\lim_{n\to\infty}\left|\frac{x-2}{n+1}\right| = 0$$

r ∞

IOC $(-\infty,\infty)$

b) $\sum_{n=1}^{\infty} \frac{3^n(x+4)^n}{\sqrt{n}}$

$$\lim_{n\to\infty}\left|\frac{3^{n+1}(x+4)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{3^n(x+4)^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{3(x+4)}{1}\right|$$

$|3(x+4)|<1$

$|x+4|<\frac{1}{3}$

Check $-3^2/_3 \to \frac{1^n}{\sqrt{n}} \to$ diverge

Check $-4^1/_3 \to \frac{(-1)^n}{\sqrt{n}} \to$ converge

R $\frac{1}{3}$

center $x=-4$

$-4^1/_3 \le x < -3^2/_3$

I.O.C

c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2n+3}}{(2n+3)!}\cdot\frac{(2n+1)!}{(-1)^n x^{2n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(-1)x^2}{(2n+3)(2n+2)}\right| = 0$$

r ∞

IOC $(-\infty,\infty)$

d) $\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$

$$\lim_{n\to\infty}\left|\frac{10^{n+1}x^{n+1}}{(n+1)^3}\cdot\frac{n^3}{10^n x^n}\right|$$

$$\lim_{n\to\infty}\left|10x\right|$$

$|10x|<1$

$|x|<1/10$

r $1/10$

IOC

$-1/10 \le x \le 1/10$

Check $x=\frac{1}{10}$ $\frac{(1)^n}{n^3} \to$ converge

Check $x=-\frac{1}{10}$ $\frac{(-1)^n}{n^3} \to$ converge

e) $\sum_{n=0}^{\infty} \frac{(-1)^n(x-3)^n}{2n+1}$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-3)^{n+1}}{2n+3}\cdot\frac{2n+1}{(-1)^n(x-3)^n}\right|$$

$$\lim_{n\to\infty}\left|(-1)^1(x-3)\right|$$

$|x-3|<1$

r: 1

$2 < x \le 4$

Check $x=2$ $\frac{(-1)^{2n}}{2n+1} \to$ Diverge

Check $x=4$ $\frac{(-1)^n}{2n+1} \to$ converge

4. Find a power series representation for the following functions and find the interval of convergence. Include the first 4 non-zero terms and the general term.

a) $f(x) = \frac{3}{1-3x} = \frac{a_1}{1-r}$ $a_1 = 3$ $r = 3x$ $\sum_{n=0}^{\infty} 3(3x)^n$ $3 + 9x + 27x^2 + 81x^3 +$

to find radius $|3x| < 1$ $-1/3 < x < 1/3$

b) $f(x) = \frac{x}{2x^2+1} = \frac{a_1}{1-r} = \frac{x}{1-(-2x^2)}$ $a_1 = x$ $r = -2x^2$ $\sum_{n=0}^{\infty} x(-2x^2)^n$

$x(-2)^n(x^2)^n$

$x - 2x^3 + 4x^5 - 8x^7 + \quad + (-2)^n x^{2n+1}$ $|-2x^2| < 1$ $|x^2| < 1/2$ IOC $-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$ $(-2)^n x^{2n+1}$

c) $f(x) = \frac{2/3}{\cancel{3}/3 - x/3} = \frac{2/3}{1-\frac{x}{3}}$ $a_1 = 2/3$ $r = x/3$

$\sum_{n=0}^{\infty} \frac{2}{3}\left(\frac{x}{3}\right)^n \to \frac{2}{3} + \frac{2x}{9} + \frac{2x^2}{27} + \frac{2x^3}{81} +$ $\left|\frac{x}{3}\right| < 1$ $|x| < 3$

$-3 < x < 3$

d) $f(x) = \frac{5}{1+x^3}$ $a_1 = 5$ $r = -x^3$ $\sum_{n=0}^{\infty} 5(-x^3)^n = 5 - 5x^3 + 5x^6 - 5x^9 + \cdots$

$\frac{5}{1+x^3} = \frac{a_1}{1-r} = \frac{5}{1-(-x^3)}$ $|-x^3| < 1$ $-1 < x < 1$

5. Find a power series representation of $f(x) = -\frac{1}{(2+x)^2}$ by using $\int \frac{-1}{(2+x)^2}dx = \frac{1}{2+x} + C$

$\frac{1}{2+x} \to \frac{1/2}{1-(-x/2)}$ $\sum_{n=0}^{\infty} \frac{1}{2}\left(-\frac{x}{2}\right)^n$

$\frac{1}{2}(-1)^n\left(\frac{1}{2}\right)^n(x)^n$

$\left(\frac{1}{2}\right)^{n+1}(-1)^n x^n$ $\to$ take derivative (w/respect to x) $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}(-1)^n n x^{n-1}$

6. Find a power series representation of $f(x) = \tan^{-1}x$ by using $f'(x) = \frac{1}{1+x^2}$.

$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$ $\sum_{n=0}^{\infty}(1)(-x^2)^n$ $(-1)^n x^{2n}$ $\int (-1)^n x^{2n} \to \frac{(-1)^n x^{2n+1}}{2n+1}$

7. The series for a function f is given by $\sum_{n=1}^{\infty}\frac{(-3)^{n-1}}{n}x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n}x^n + \cdots$ and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence.

a) Use the ratio test to find R.

$\lim_{n\to\infty}\left|\frac{(-3)^n x^{n+1}}{n+1}\cdot\frac{n}{(-3)^{n-1}x^n}\right| \to \lim_{n\to\infty}|-3x|$ $|-3x| < 1$

C O $R = 1/3$ IOC $-1/3 < x \leq 1/3$

Check $x = -1/3$ diverge

Check $x = 1/3$ converge

b) Write the first four nonzero terms of the series for f', the derivative of f. Express f' as a rational function for $|x| < R$.

Write the first (4) terms of $f \to x - \frac{3}{2}x^2 + 3x^3 - \frac{27}{4}x^4$

$f' \to 1 - 3x + 9x^2 - 27x^3$

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rational function for f' $\to \frac{1}{1+3x} \to$ rational function for $f = \frac{1}{3}\ln|1+3x|$ (integral!)