$X \sim B(n, p)$ $n \rightarrow$ # of trials p - probability of success

Binomial and Normal Distribution Notes Name:______________________________

$X \sim B(6, \frac{1}{5})$

1. X is binomially distributed with 6 trials and a probability of success equal to $\frac{1}{5}$ at each attempt. What is the probability of

   a. Exactly 4 successes

   $\binom{6}{4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2$ or binompdf$(6, \frac{1}{5}, 4)$

   $= 01536$

   b. At least 1 success

   $1 - P(\text{none}) \longrightarrow 1 - 26144 = \boxed{738}$

   $P(\text{None}) = \binom{6}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6$

   $= 262144$

   c. 3 or fewer successes

   instead of doing $P(0)+P(1)+P(2)+P(3)$ $\rightarrow$ binomcdf$(6, \frac{1}{5}, 3)$

   $= 98304$

2. If $X \sim B(6, \frac{1}{3})$ find to 3 significant figures

   a. $P(X = 2)$

   binompdf$(6, \frac{1}{3}, 2) = 329$

   b. $P(X < 2)$

   binomcdf$(6, \frac{1}{3}, 1) = 351$

   c. $P(X \leq 2)$

   680

   d. $P(X \geq 2)$

   $1 - P(0 \text{ or } 1) = 1 - 351 = .649$

3. The probability that I get a bus to work on any morning is 0.4. What is the probability that in a working week of five days I will get a bus twice?

success

$$\binom{5}{2}(4)^{2}(6)^{3} = 3456$$

4. A box contains a large number of carnations of which one-quarter are red. The rest are white. Carnations are picked at random from the box. How many flowers must be picked so that the probability that there is as least one red carnation among them is greater than 0.95.

P(At least 1 red) = 1 − P(no red)

$P(\text{no red}) = \binom{x}{0}(25)^{0}(75)^{x}$

$1 \quad 1 \quad 75^{x}$

$P(\text{No red}) = 75^{x}$

$$1 - 75^{x} > 95$$
$$-75^{x} > -05$$
$$75^{x} < 05$$
$$x \ln(.75) < \ln(05)$$
$$x > \frac{\ln(05)}{\ln(75)}$$

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5. A biased dice is thrown 30 times and the number of sixes is 8. The dice is then thrown a further 12 times. Find the expected number of sixes for these 12 throws.

$E(x) = n \cdot p$

$\boxed{E(x) \rightarrow \text{mean}}$

$p \rightarrow \frac{8}{30}$

$n \rightarrow 12$

$12\left(\frac{8}{30}\right) \rightarrow 3\ 2$

6. X is a random variable such that $\boxed{X \sim B(n,p)}$. Given that the mean of the distribution is 10 and $p = 0.4$, find n.

Expected value

$10 = n(4)$

$n = \frac{10}{4} = 25$

7. In a large company, 40% of the workers travel to work on public transport. A random sample of 15 workers is selected. ① Find the expected number of workers in this sample that travel to work on public transport and the ② standard deviation.

$\boxed{\text{variance} = npq}$ $\quad q = 1 - p$

① $(15)(.40) = \boxed{6}$

② $(15)(.40)(.60) = 6(.6) = 3\ 6 = \text{variance}$ $\quad \boxed{190}$

Stand dev $= \sqrt{\text{variance}} = \sqrt{3.6}$

$X \sim N(\mu, \sigma^2)$ Standardized normal

8. Given that $Z \sim N(0,1)$, find

$Z \sim N(0,1)$

a. $P(-2 < Z < 1)$

3415
+.3415
+1355

8185

normalcdf(-2, 1, 0, 1)
= 8186

b. $P(Z < 1)$

50
+ 3415

$\boxed{8415}$

In calc
normalcdf(-1E99, 1, 0, 1)
$\boxed{= 8413}$

c. $P(Z > -1.5)$

normalcdf(-1 5, 1E99, 0, 1)

933

d. $P(Z < 0)$

.50

e. $P(|Z| > 0.8)$

P(Z>0 8) + P(Z<-0.8)

212 + 212 = 424

f. $P(|Z| < 0.4)$

P(-4<Z< 4) = .311

$$Z = \frac{X - \mu}{\sigma}$$

9. The random variable $X \sim N(10, 4)$. Find $P(9.1 < X < 10.3)$

$\mu \quad \sigma^2 \to \sigma = 2$

$Z \sim N(0,1)$

$$Z = \frac{9.1 - 10}{2} = \frac{-.9}{2} = -.45$$

$$Z = \frac{10.3 - 10}{2} = \frac{.3}{2} = .15$$

$P(-.45 < Z < .15)$

$\boxed{= .233}$

10. The random variable $X \sim N(48, 81)$.

$\mu \quad \sigma^2 \to \sigma \to 9$

a. Find $P(X < 52)$

$$Z = \frac{52 - 48}{9} = \frac{4}{9}$$

$\approx .672$

b. Find $P(X \geq 42)$

$\approx .748$

c. Find $P(37 < X < 47)$

$\approx .345$

11. Eggs laid by a chicken have mass normally distributed, with mean 55 g and standard deviation 2.5 g. What is

a. Probability that an egg weighs more than 59 g

b. Probability that an egg weighs between 52 g and 54 g

c. Probability than an egg weighs less than 53 g