Calculus Test Review Name:______________________

1a. *[3 marks]*

Let $y = \left(x^3 + x\right)^{\frac{3}{2}}$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

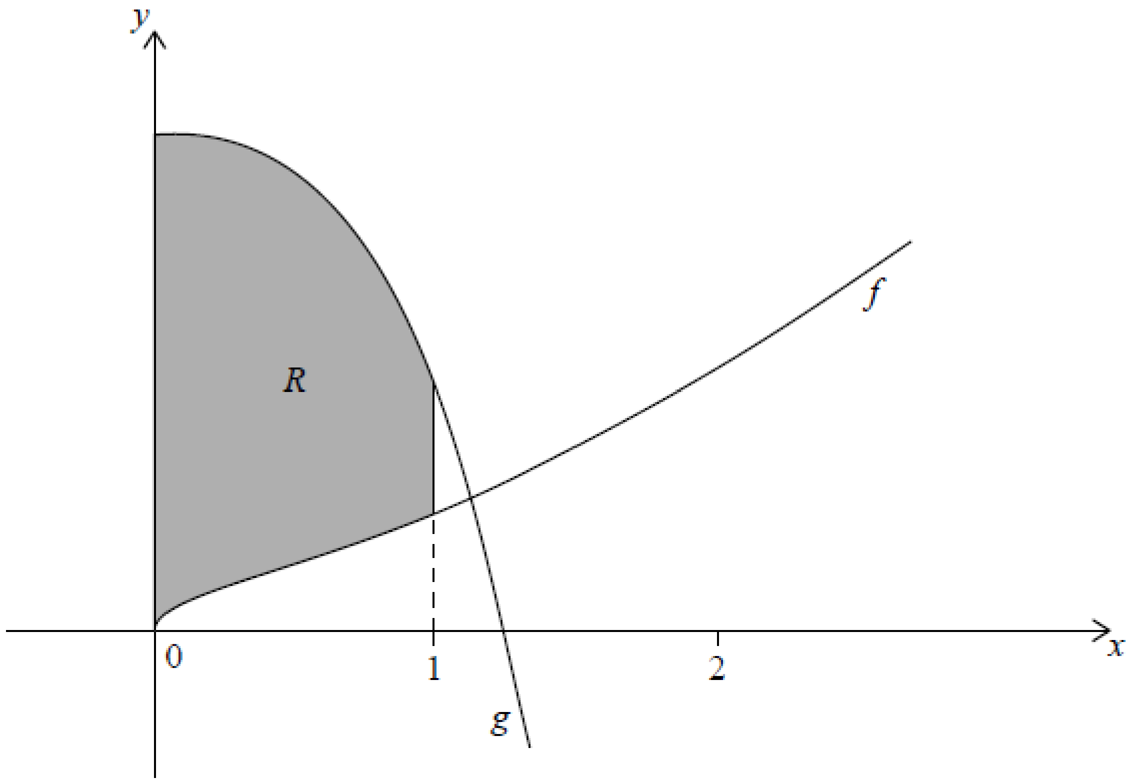
1b. *[3 marks]*

Hence find $\int \left(3x^2 + 1\right)\sqrt{x^3 + x}\,\mathrm{d}x$.

1c. *[2 marks]*

Consider the functions $f\left(x\right) = \sqrt{x^3 + x}$ and $g\left(x\right) = 6 - 3x^2\sqrt{x^3 + x}$, for $x \geq 0$.

The graphs of f and g are shown in the following diagram.



The shaded region R is enclosed by the graphs of f, g, the y-axis and $x = 1$.

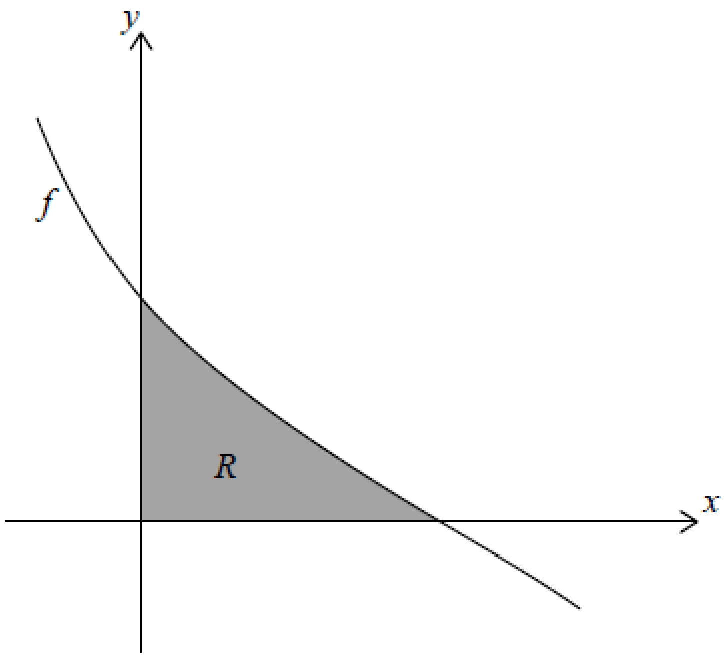
Write down an expression for the area of R.

1d. *[6 marks]*

Hence find the exact area of R.

2. *[8 marks]*

Let $f\left(x\right) = \frac{6-2x}{\sqrt{16+6x-x^2}}$. The following diagram shows part of the graph of f.



The region *R* is enclosed by the graph of f, the x-axis, and the y-axis. Find the area of *R*.

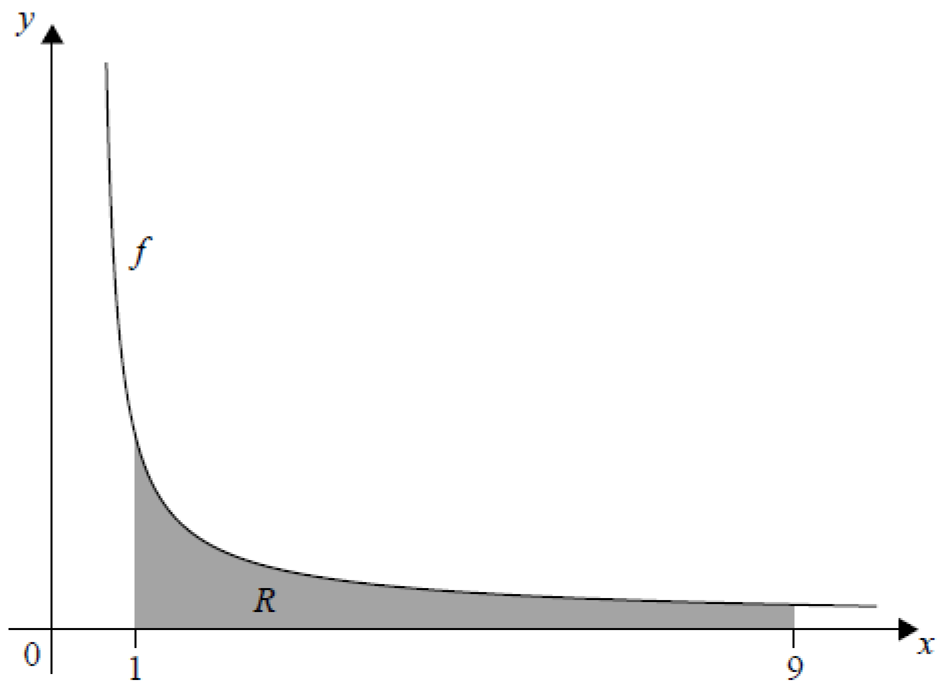
3a. *[3 marks]*

Let $f\left(x\right) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

Find $\int \left(f\left(x\right)\right)^2 \mathrm{d}x$.

3b. *[4 marks]*

Part of the graph of *f* is shown in the following diagram.



The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = 9$. Find the volume of the solid formed when R is revolved 360° about the x-axis.

4. *[7 marks]*

Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.

5a. *[6 marks]*

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x-intercept at $x = -1$.

Find $f(x)$.

5b. *[4 marks]*

The graph of f has a point of inflexion at $x = p$. Find p.

5c. *[3 marks]*

Find the values of x for which the graph of f is concave-down.

6a. *[2 marks]*

Let $f(x) = \frac{16}{x}$. The line L is tangent to the graph of f at $x = 8$.

Find the gradient of L.

6b. *[2 marks]*

L can be expressed in the form $\boldsymbol{r} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} + t\boldsymbol{u}$.

Find $\boldsymbol{u}$.

6c. *[5 marks]*

The direction vector of $y = x$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Find the acute angle between $y = x$ and L.

6d. *[3 marks]*

Find $(f \circ f)(x)$.

6e. *[1 mark]*

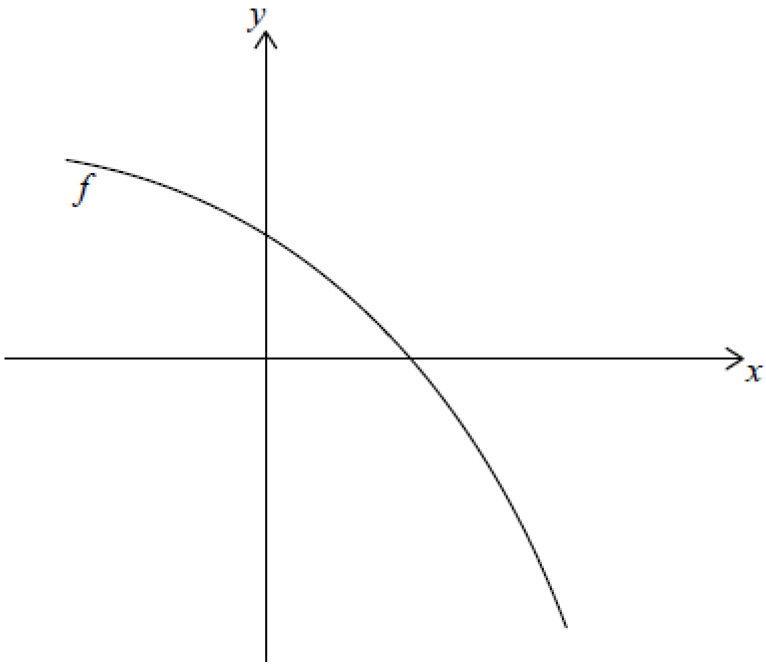
Hence, write down $f^{-1}(x)$.

6f. *[3 marks]*

Hence or otherwise, find the obtuse angle formed by the tangent line to f at $x = 8$ and the tangent line to f at $x = 2$.

7a. *[2 marks]*

Let $f(x) = 4 - 2\mathrm{e}^x$. The following diagram shows part of the graph of f.



Find the x-intercept of the graph of f.

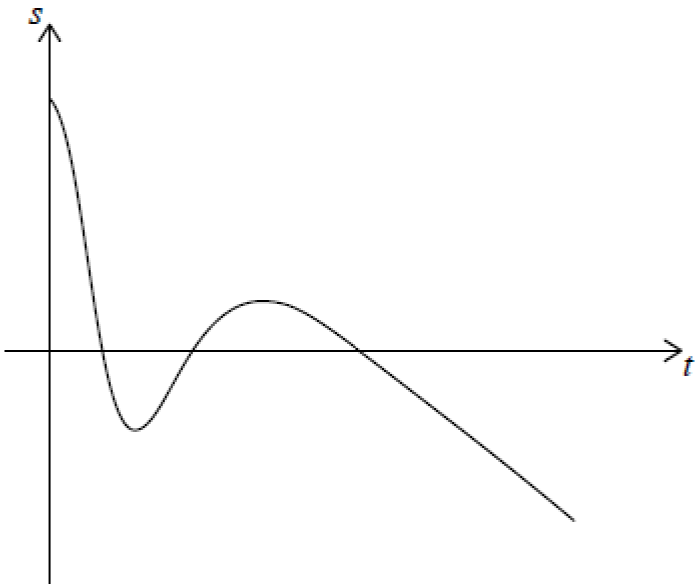
7b. *[3 marks]*

The region enclosed by the graph of f, the x-axis and the y-axis is rotated 360º about the x-axis. Find the volume of the solid formed.

8a. *[2 marks]*

In this question distance is in centimetres and time is in seconds.

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_{\mathrm{A}} = 15 - t - 6t^3\mathrm{e}^{-0.8t}$, $0 \le t \le 25$. This is shown in the following diagram.



Find the initial displacement of particle A from point P.

8b. *[2 marks]*

Find the value of t when particle A first reaches point P.

8c. *[2 marks]*

Find the value of t when particle A first changes direction.

8d. *[3 marks]*

Find the total distance travelled by particle A in the first 3 seconds.

8e. *[5 marks]*

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_B = 8 - 2t$, $0 \le t \le 25$.

Given that particles A and B start at the same point, find the displacement function s_B for particle B.

8f. *[2 marks]*

Find the other value of t when particles A and B meet.