Calculus Test Review *[86 marks]*

1a. *[3 marks]*

Markscheme

evidence of choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x},\ u=x^3+x,\ u'=3x^2+1$

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}\left(x^3+x\right)^{\frac{1}{2}}\left(3x^2+1\right)\ \left(=\frac{3}{2}\sqrt{x^3+x}\left(3x^2+1\right)\right)$ ***A2 N3***

[3 marks]

1b. *[3 marks]*

Markscheme

integrating by inspection from (a) or by substitution ***(M1)***

eg $\frac{2}{3}\int\frac{3}{2}\left(3x^2+1\right)\sqrt{x^3+x}\,\mathrm{d}x,\ u=x^3+x,\ \frac{\mathrm{d}u}{\mathrm{d}x}=3x^2+1,\ \int u^{\frac{1}{2}},\ \frac{u^{\frac{3}{2}}}{1.5}$

correct integrated expression in terms of x ***A2 N3***

eg $\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}+C,\ \frac{\left(x^3+x\right)^{1.5}}{1.5}+C$

[3 marks]

1c. *[2 marks]*

Markscheme

integrating and subtracting functions (in any order) ***(M1)***

eg $\int g - f,\ \int f - \int g$

correct integral (including limits, accept absence of $\mathrm{d}x$) ***A1 N2***

eg $\int_0^1 (g - f)\,\mathrm{d}x,\ \int_0^1 6 - 3x^2\sqrt{x^3+x} - \sqrt{x^3+x}\,\mathrm{d}x,\ \int_0^1 g(x) - \int_0^1 f(x)$

[2 marks]

1d. *[6 marks]*

Markscheme

recognizing $\sqrt{x^3+x}$ is a common factor (seen anywhere, may be seen in part (c)) ***(M1)***

eg $\left(-3x^2-1\right)\sqrt{x^3+x},\ \int 6 - \left(3x^2+1\right)\sqrt{x^3+x},\ \ \left(3x^2-1\right)\sqrt{x^3+x}$

correct integration ***(A1)(A1)***

eg $6x - \frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$

Note: Award ***A1*** for $6x$ and award ***A1*** for $-\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $6 - \frac{2}{3}\left(1^3+1\right)^{\frac{3}{2}},\ 0 - \left[6 - \frac{2}{3}\left(1^3+1\right)^{\frac{3}{2}}\right]$

correct working ***(A1)***

eg $6 - \frac{2}{3} \times 2\sqrt{2},\ 6 - \frac{2}{3} \times \sqrt{4} \times \sqrt{2}$

area of $R = 6 - \frac{4\sqrt{2}}{3}\ \left(= 6 - \frac{2}{3}\sqrt{8},\ 6 - \frac{2}{3} \times 2^{\frac{3}{2}},\ \frac{18-4\sqrt{2}}{3}\right)$ ***A1 N3***

[6 marks]

2. *[8 marks]*

Markscheme

METHOD 1 (limits in terms of x)

valid approach to find x-intercept ***(M1)*** *eg* $f(x)=0,\ \frac{6-2x}{\sqrt{16+6x-x^2}}=0,\ 6-2x=0$

x-intercept is 3 ***(A1)***

valid approach using substitution or inspection ***(M1)***

eg $u=16+6x-x^2,\ \int_0^3 \frac{6-2x}{\sqrt{u}}\mathrm{d}x,\ \mathrm{d}u=6-2x,\ \int \frac{1}{\sqrt{u}},\ 2u^{\frac{1}{2}},$

$u=\sqrt{16+6x-x^2},\ \ \frac{\mathrm{d}u}{\mathrm{d}x}=(6-2x)\,\frac{1}{2}\left(16+6x-x^2\right)^{-\frac{1}{2}},\ \int 2\,\mathrm{d}u,\ 2u$

$\int f(x)\,\mathrm{d}x=2\sqrt{16+6x-x^2}$ ***(A2)***

substituting **both** of **their** limits into **their** integrated function and subtracting ***(M1)***

eg $2\sqrt{16+6(3)-3^2}-2\sqrt{16+6(0)^2-0^2},\ 2\sqrt{16+18-9}-2\sqrt{16}$

Note: Award ***M0*** if they substitute into original or differentiated function. Do not accept only "– 0" as evidence of substituting lower limit.

correct working ***(A1)***

eg $2\sqrt{25}-2\sqrt{16},\ 10-8$

area = 2 ***A1 N2***

METHOD 2 (limits in terms of u)

valid approach to find x-intercept ***(M1)***

eg $f(x)=0,\ \frac{6-2x}{\sqrt{16+6x-x^2}}=0,\ 6-2x=0$

x-intercept is 3 ***(A1)***

valid approach using substitution or inspection ***(M1)***

eg $u=16+6x-x^2,\ \int_0^3 \frac{6-2x}{\sqrt{u}}\mathrm{d}x,\ \mathrm{d}u=6-2x,\ \int \frac{1}{\sqrt{u}},$

$u=\sqrt{16+6x-x^2},\ \ \frac{\mathrm{d}u}{\mathrm{d}x}=(6-2x)\,\frac{1}{2}\left(16+6x-x^2\right)^{-\frac{1}{2}},\ \int 2\,\mathrm{d}u$

correct integration ***(A2)***

eg $\int \frac{1}{\sqrt{u}}\,\mathrm{d}u=2u^{\frac{1}{2}},\ \ \int 2\,\mathrm{d}u=2u$

both correct limits for u ***(A1)***

eg u = 16 and u = 25, $\int_{16}^{25} \frac{1}{\sqrt{u}}\mathrm{d}u,\ \ \left[2u^{\frac{1}{2}}\right]_{16}^{25},$ u = 4 and u = 5, $\int_4^5 2\,\mathrm{d}u,$ $[2u]_4^5$

substituting **both** of **their** limits for u (do not accept 0 and 3) into **their** integrated function and subtracting ***(M1)***

eg $2\sqrt{25}-2\sqrt{16},\ 10-8$

Note: Award ***M0*** if they substitute into original or differentiated function, or if they have not attempted to find limits for u.

area = 2 ***A1 N2***

[8 marks]

3a. *[3 marks]*

Markscheme

correct working ***(A1)***

eg $\int \frac{1}{2x-1}\mathrm{d}x,\ \int (2x-1)^{-1},\ \frac{1}{2x-1},\ \int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2 \mathrm{d}x = \frac{1}{2}\ln(2x-1)+c$ ***A2 N3***

Note: Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

3b. *[4 marks]*

Markscheme

attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / dx) ***(M1)***

eg $\int_1^9 y^2 \mathrm{d}x,\ \pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2 \mathrm{d}x,\ \left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

eg $\frac{\pi}{2}(\ln(17)-\ln(1)),\ \pi\left(0-\frac{1}{2}\ln(2\times 9-1)\right)$

correct working involving calculating a log value or using log law ***(A1)***

eg $\ln(1)=0,\ \ln\left(\frac{17}{1}\right)$

$\frac{\pi}{2}\ln 17\ \ \left(\text{accept } \pi\ln\sqrt{17}\right)$ ***A1 N3***

Note: Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

[4 marks]

4. *[7 marks]*

Markscheme

recognizing the need to find h' ***(M1)***

recognizing the need to find $h'(3)$ (seen anywhere) ***(M1)***

evidence of choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x},\ f'(g(3))\times g'(3),\ f'(g)\times g'$

correct working ***(A1)***

eg $f'(7)\times 4,\ -5\times 4$

$h'(3)=-20$ ***(A1)***

evidence of taking **their** negative reciprocal for normal ***(M1)***

eg $-\frac{1}{h'(3)},\ m_1m_2=-1$

gradient of normal is $\frac{1}{20}$ ***A1 N4***

[7 marks]

5a. *[6 marks]*

Markscheme

evidence of integration ***(M1)***

eg $\int f'\left(x\right)$

correct integration (accept absence of *C*) ***(A1)(A1)***

eg $x^3+\frac{18}{2}x^2+C,\ x^3+9x^2$

attempt to substitute $x = -1$ into **their** $f = 0$ (must have *C*) ***M1***

eg $\left(-1\right)^3+9\left(-1\right)^2+C=0,\ -1+9+C=0$

Note: Award ***M0*** if they substitute into original or differentiated function.

correct working ***(A1)***

eg $8+C=0,\ \ C=-8$

$f\left(x\right)=x^3+9x^2-8$ ***A1 N5***

[6 marks]

5b. *[4 marks]*

Markscheme

METHOD 1 (using 2nd derivative)

recognizing that $f'' = 0$ (seen anywhere) ***M1***

correct expression for f'' ***(A1)***

eg $6x + 18$, $6p + 18$

correct working ***(A1)***

$6p + 18 = 0$

$p = -3$ ***A1 N3***

METHOD 1 (using 1st derivative)

recognizing the vertex of f' is needed ***(M2)***

eg $-\frac{b}{2a}$ (must be clear this is for f')

correct substitution ***(A1)***

eg $\frac{-18}{2\times3}$

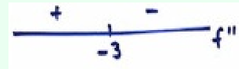
$p = -3$ ***A1 N3***

[4 marks]

5c. *[3 marks]*

Markscheme

valid attempt to use $f''(x)$ to determine concavity ***(M1)***

eg $f''(x) < 0$, $f''(-2)$, $f''(-4)$, $6x + 18 \leq 0$ 

correct working ***(A1)***

eg $6x + 18 < 0$, $f''(-2) = 6$, $f''(-4) = -6$ 

f concave down for $x < -3$ (do not accept $x \leq -3$) ***A1 N2***

[3 marks]

6a. *[2 marks]*

Markscheme

attempt to find $f'(8)$ ***(M1)***

eg $f'(x)$, y' , $-16x^{-2}$

-0.25 (exact) ***A1 N2***

[2 marks]

6b. *[2 marks]*

Markscheme

$\boldsymbol{u} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ or any scalar multiple ***A2 N2***

[2 marks]

6c. *[5 marks]*

Markscheme

correct scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $= 1 \times 4 + 1 \times -1 \ \ (= 3)$

magnitudes $= \sqrt{1^2 + 1^2}, \ \sqrt{4^2 + (-1)^2} \ \left(= \sqrt{2}, \sqrt{17}\right)$

substitution of their values into correct formula ***(M1)***

eg $\frac{4-1}{\sqrt{1^2+1^2}\sqrt{4^2+(-1)^2}}, \ \frac{-3}{\sqrt{2}\sqrt{17}}$, 2.1112, 120.96°

1.03037 , 59.0362°

angle = 1.03 , 59.0° ***A1 N4***

[5 marks]

6d. *[3 marks]*

Markscheme

attempt to form composite $(f \circ f)(x)$ ***(M1)***

eg $f(f(x))$, $f\left(\frac{16}{x}\right)$, $\frac{16}{f(x)}$

correct working ***(A1)***

eg $\frac{16}{\frac{16}{x}}$, $16 \times \frac{x}{16}$

$(f \circ f)(x) = x$ ***A1 N2***

[3 marks]

6e. *[1 mark]*

Markscheme

$f^{-1}(x) = \frac{16}{x}$ (accept $y = \frac{16}{x}, \frac{16}{x}$) ***A1 N1***

Note: Award ***A0*** in part (ii) if part (i) is incorrect.
Award ***A0*** in part (ii) if the candidate has found $f^{-1}(x) = \frac{16}{x}$ by interchanging x and y.

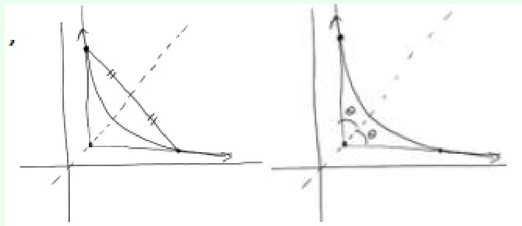
[1 mark]

6f. *[3 marks]*

Markscheme

METHOD 1

recognition of symmetry about $y = x$ ***(M1)***

eg (2, 8) ⇔ (8, 2) 

evidence of doubling **their** angle ***(M1)***

eg $2 \times 1.03,\ 2 \times 59.0$

2.06075, 118.072°

2.06 (radians) (118 degrees) ***A1 N2***

METHOD 2

finding direction vector for tangent line at $x = 2$ ***(A1)***

eg $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

substitution of **their** values into correct formula (must be from vectors) ***(M1)***

eg $\frac{-4-4}{\sqrt{1^2+4^2}\sqrt{4^2+(-1)^2}}$, $\frac{8}{\sqrt{17}\sqrt{17}}$

2.06075, 118.072°

2.06 (radians) (118 degrees) ***A1 N2***

METHOD 3

using trigonometry to find an angle with the horizontal ***(M1)***

eg $\tan\theta = -\frac{1}{4},\ \tan\theta = -4$

finding both angles of rotation ***(A1)***

eg $\theta_1 = 0.244978,\ 14.0362°,\ \theta_1 = 1.81577,\ 104.036°$

2.06075, 118.072°

2.06 (radians) (118 degrees) ***A1 N2***

[3 marks]

7a. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $f(x) = 0, \quad 4 - 2e^{x} = 0$

0.693147

x = ln 2 (exact), 0.693 ***A1 N2***

[2 marks]

7b. *[3 marks]*

Markscheme

attempt to substitute either their correct limits or the function into formula ***(M1)***

involving f^2

eg $\int_0^{0.693} f^2$, $\pi \int (4 - 2e^{x})^2 dx, \quad \int_0^{\ln 2} (4 - 2e^{x})^2$

3.42545

volume = 3.43 ***A2 N3***

[3 marks]

8a. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $s_A(0), s(0), t = 0$

15 (cm) ***A1 N2***

[2 marks]

8b. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $s_{\mathrm{A}}=0,\ s=0,\ 6.79321,\ 14.8651$

2.46941

t = 2.47 (seconds) ***A1 N2***

[2 marks]

8c. *[2 marks]*

Markscheme

recognizing when change in direction occurs ***(M1)***

eg slope of s changes sign, $s'=0$, minimum point, 10.0144, (4.08, −4.66)

4.07702

t = 4.08 (seconds) ***A1 N2***

[2 marks]

8d. *[3 marks]*

Markscheme

METHOD 1 (using displacement)

correct displacement or distance from P at $t = 3$ (seen anywhere) ***(A1)***

eg −2.69630, 2.69630

valid approach ***(M1)***

eg 15 + 2.69630, $s(3) - s(0)$, −17.6963

17.6963

17.7 (cm) ***A1 N2***

METHOD 2 (using velocity)

attempt to substitute either limits or the velocity function into distance formula involving $|v|$ ***(M1)***

eg $\int_0^3 |v|\,\mathrm{d}t$, $\int \left|-1 - 18t^2\mathrm{e}^{-0.8t} + 4.8t^3\mathrm{e}^{-0.8t}\right|$

17.6963

17.7 (cm) ***A1 N2***

[3 marks]

8e. *[5 marks]*

Markscheme

recognize the need to integrate velocity ***(M1)***

eg $\int v(t)$

$8t - \frac{2t^2}{2} + c$ (accept x instead of t and missing c) ***(A2)***

substituting initial condition into their integrated expression (must have c) ***(M1)***

eg $15 = 8(0) - \frac{2(0)^2}{2} + c$, $c = 15$

$s_{\mathrm{B}}(t) = 8t - t^2 + 15$ ***A1 N3***

[5 marks]

8f. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $s_{\mathrm{A}} = s_{\mathrm{B}}$, sketch, (9.30404, 2.86710)

9.30404

$t = 9.30$ (seconds) ***A1 N2***

Note: If candidates obtain $s_{\mathrm{B}}(t) = 8t - t^2$ in part (e)(i), there are 2 solutions for part (e)(ii), 1.32463 and 7.79009. Award the last ***A1*** in part (e)(ii) only if both solutions are given.

[2 marks]