AB Calculus The Chain Rule Part 2 Homework Name: ______________________

1. Find dy/dx.

$\frac{4}{5}x^{\frac{4}{5}}$ $3x^{-2}$

a) $y = -2x^3 + \frac{4}{5\sqrt[5]{x^4}} - \frac{3}{x^2}$

$-6x^2 - \frac{16}{25x\sqrt[5]{x^4}} + \frac{6}{x^3}$

b) $f(x) = \frac{2t}{2+\sqrt{t}}$

$\frac{(2+\sqrt{t})2 - 2t\left(\frac{1}{2\sqrt{t}}\right)}{(2+\sqrt{t})^2}$

$\frac{4+2\sqrt{t} - \frac{2t\sqrt{t}}{2\sqrt{t}\sqrt{t}}}{(2+\sqrt{t})^2} = \frac{4+2\sqrt{t}-\sqrt{t}}{(2+\sqrt{t})^2}$

$= \frac{4+\sqrt{t}}{(2+\sqrt{t})^2}$

c) $y = \csc\theta\,(\theta + \cot\theta)$

$-\csc\theta\cot\theta(\theta+\cot\theta)$
$+\csc\theta(1-\csc^2\theta)$

$= -\theta\csc\theta\cot\theta - \csc\theta\cot^2\theta + \csc\theta - \csc^3\theta$

d) $y = \frac{\sin x}{x^3}$

$\frac{x^3\cos x - \sin x(3x^2)}{x^6}$

e) $y = \sin x \tan x$

$\cos x\tan x + \sin x\sec^2 x$

f) $y = \tan^2(3\theta)$

$(\tan 3\theta)^2$

$= 2\tan 3\theta\sec^2(3\theta)3$

$= 6\tan(3\theta)\sec^2(3\theta)$

g) $y = \sin(\sin(\sin x))$

$\sin x \to \cos x$ $\cos(\sin(\sin x))\cdot\cos(\sin x)\cdot\cos x$

$\sin x \to \cos x$

$\sin x \to \cos x$

h) $y = \cos^2\sqrt{\sin(\tan(\pi x))}$

$x^2 \to 2x$
$\cos x \to -\sin x$
$\sqrt{x} \to \frac{1}{2\sqrt{x}}$
$\sin x \to \cos x$
$\tan(\pi x) \to \pi\sec^2(\pi x)$

$2\cos\sqrt{\sin(\tan\pi x)}\cdot -\sin\sqrt{\sin(\tan\pi x)}$
$\cdot\frac{1}{2\sqrt{\sin(\tan\pi x)}}\cdot\cos(\tan\pi x)$
$\cdot\pi\sec^2(\pi x)$

2. If $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, $g'(2) = 7$, find $h'(2)$ for each of the following.

a) $h(x) = 5f(x) - 4g(x)$

$5f'(x) - 4g'(x)$
$5(-2) - 4(7)$
$= -10 - 28 = -38$

b) $h(x) = f(x)g(x)$

$f'(x)g(x) + g'(x)f(x)$
$(-2)(4) + (7)(-3) = -8 - 21 = -29$

c) $h(x) = \frac{f(x)}{g(x)}$

$\frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

$\frac{(4)(-2)-(-3)(7)}{16} = \frac{-8+21}{16} = \frac{13}{16}$

d) $h(x) = \frac{g(x)}{1+f(x)}$

$\frac{(1+f(x))g'(x) - g(x)f'(x)}{(1+f(x))^2}$

Numerator: $(1+-3)(7) - (4)(-2)$
$(-2)(7) + 8$
$-14 + 8$
-6

Den: $(1+-3)^2$
$(-2)^2$
$= 4$

$\frac{-6}{4} = -\frac{3}{2}$

3. Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

$f(x) \to f'(x)$ $\quad f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$ $\quad h(1) = 2$ $\quad h'(1) = 4$

$g(x) \to g'(x)$ $\quad g'(2) = 5$

$h(x) \to h'(x)$ $\quad g(h(x)) = g(h(1)) = g(2) = 3$ $\quad f'(3) = 6$ $\quad 6 \cdot 5 \cdot 4 = 120$

4. Given the table of values for f, g, f', and g', find the following values.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a) If $h(x) = f(g(x))$ find $h'(1)$

$f'(g(1)) \cdot g'(1)$

$f'(2) = 5$ $\quad 5 \cdot 6$

$g'(1) = 6$ $\quad = 30$

b) If $h(x) = g(f(x))$ find $h'(1)$

$g'(f(1)) \cdot f'(1)$

$g'(3) = 9$ $\quad 9 \cdot 4 = 36$

$f'(1) = 4$

c) If $h(x) = f(f(x))$ find $h'(2)$

$f'(f(2)) \cdot f'(2)$ $\quad f'(2) = 5$ $\quad 4 \cdot 5 = 20$

$f(2) = 1$ $\quad f'(1) = 4$

d) If $h(x) = g(g(x))$ find $h'(3)$

$g'(g(x)) \cdot g'(x)$

$g'(g(3)) = g'(2) = 7 \to 7 \cdot 9$

$g'(3) = 9$ $\quad = 63$

5. Find the equation of the tangent line to the curve $y = \sin x + \sin^2 x$ at the point $(0, 0)$.

$\sin 0 + \sin^2 0 = 0$

$\cos x + 2\sin x \cos x @ 0$

$\cos 0 + 2\sin 0 \cdot \cos 0 = 1 + 2 \cdot 0 \cdot 1 = 1$ $\quad y - 0 = 1(x - 0) \to y = x$

6. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in meters.

a) Find the velocity of the particle at time $t = 2$. Indicate units of measure.

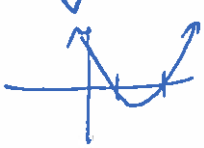
$3t^2 - 12t + 9 \to 3(4) - 24 + 9 = 12 - 24 + 9 = -12 + 9 = -3\ m/s$

b) When is the particle at rest? Justify your response.

$\frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3} = \frac{0}{3}$ $\quad t^2 - 4t + 3 = 0$ $\quad t = 1, 3$

$(t - 1)(t - 3) = 0$

c) Determine whether the particle is speeding up or slowing down at time $t = 4$. Justify your response.



speeding up: $(1, 2)$ $\quad$ slowing down $(-\infty, 1)$

$(3, \infty)$ $\quad (2, 3)$

7. Suppose a company has estimated the cost in dollars of producing x items is $C(x) = 10000 + 5x + .01x^2$. Predict the cost of producing the 501st item.

$5 + .02x$ $\quad 5 + \frac{500}{50} = 5 + 10.00$

$5 + .02(50) =$ $\quad = \$15.00$