AP Calculus Name________________________________

Chain Rule, Deriving Trig Functions and Position Notes

Date________________

Differentiate each function with respect to the given variable.

1) $h = \sqrt{\frac{5t^2 + 2}{3t^4 + 5}}$

$\frac{d}{dt}\left(\frac{5t^2+2}{3t^4+5}\right)^{\frac{1}{2}}$

Find $h'(t)$ or $\frac{dh}{dt}$

$$= \boxed{\frac{1}{2}\left(\frac{5t^2+2}{3t^4+5}\right)^{-\frac{1}{2}}\left(\frac{(3t^4+5)(10t)-(5t^2+2)(12t^3)}{(3t^4+5)^2}\right)}$$

2) $g = (-s^2 - 5)^{-3}\sqrt{3s^3 - 5}$

$\sqrt{3s^3-5} \rightarrow (3s^3-5)^{\frac{1}{2}}$

$g'(s)$ or $\frac{dg}{ds} \rightarrow \underbrace{(-s^2-5)^{-3}}_{1^{st}} \underbrace{\frac{1}{2}(3s^3-5)^{-\frac{1}{2}}(9s^2)}_{\text{Derivative of 2nd}} \underset{\text{Formula says +}}{+} \underbrace{(-3)(-s^2-5)^{-4}(-2s)}_{\text{D of 1st}} \underbrace{\sqrt{3s^3-5}}_{\text{2nd}}$

3) $h(s) = (4s^3 + 1)^{-3}$

$h'(s) = -3(4s^3+1)^{-4}(12s^2)$ simplify $\rightarrow \frac{-36s^2}{(4s^3+1)^4}$

or $\frac{dh}{ds}$

4) $f = \left((3x+5)^3 - 2\right)^{\frac{1}{4}}$

Outs.

$$x^{\frac{1}{4}} \xrightarrow{\frac{d}{dx}} \boxed{\frac{1}{4}x^{-\frac{3}{4}}}$$

$$x^3 - 2 \rightarrow \boxed{3x^2}$$

$$3x+5 \rightarrow \boxed{3}$$

Inside

$$\boxed{f'(x) = \frac{1}{4}\left((3x+5)^3 - 2\right)^{-\frac{3}{4}} \cdot 3(3x+5)^2\, 3}$$

alt

$$x^{\frac{1}{4}} \rightarrow \frac{1}{4}x^{-\frac{3}{4}}$$

$$x - 2 \rightarrow 1$$

$$x^3 \rightarrow 3x^2$$

$$3x+5 \rightarrow 3$$

$$\frac{1}{4}\left((3x+5)^3 - 2\right)^{-\frac{3}{4}} \mid 3(3x+5)^2 \; 3$$

Differentiate each function with respect to *x*.

5) $f(x) = \dfrac{\csc(4x^4)}{\cot(x^3)}$

derivative of numerator

chain rule

chain rule

$$f'(x) = \frac{\cot(x^3) \cdot -\csc(4x^4)\cot(4x^4) \cdot 16x^3 - \csc(4x^4)\left(-\csc^2(x^3)\, 3x^2\right)}{\cot^2(x^3)}$$

6) $f(x) = \cot(\sin 3x^3)$

Outside

$$\cot x \xrightarrow{\frac{d}{dx}} -\csc^2 x$$

$$\sin x \rightarrow \cos x$$

$$3x^3 \rightarrow 9x^2$$

Inside

$$f'(x) = -\csc^2(\sin 3x^3) \cdot \cos(3x^3) \cdot 9x^2$$

simplify $-9x^2\csc^2(\sin(3x^3))\cos(3x^3)$

7) $y = \frac{-16x}{(2x-5)^2}$

$$\frac{dy}{dx} = \frac{(2x-5)^2(-16) - (-16x)\cdot 2(2x-5)\,2}{(2x-5)^4}$$

simplify $\frac{(2x-5)(-16) + 64x}{(2x-5)^3} \rightarrow \frac{-32x+80+64x}{(2x-5)^3} \rightarrow \boxed{\frac{32x+80}{(2x-5)^3}}$

For each problem, find the equation of the line tangent to the function at the given point.

8) $y = \frac{-16x}{(2x-5)^2}$ at $x = 2$

$x, y, \frac{dy}{dx} \rightarrow$ From problem 7

$x \downarrow 2$

$\left.\frac{dy}{dx}\right|_{x=2} = \frac{32\cdot 2+80}{(2\cdot 2-5)^3} = \frac{144}{-1} = -144$

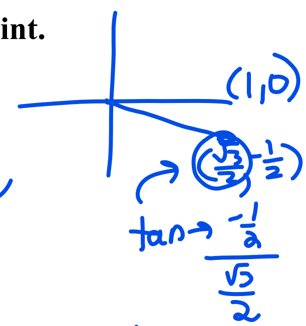
$y(2) = \frac{-16(2)}{(2\cdot 2-5)^2} \rightarrow -32$

$\boxed{y+32 = -144(x-2)}$

For each problem, find the equation of the line normal to the function at the given point.

9) $y = -\sec(x)$ at $x = -\frac{\pi}{6}$

$x, y, \frac{dy}{dx}$



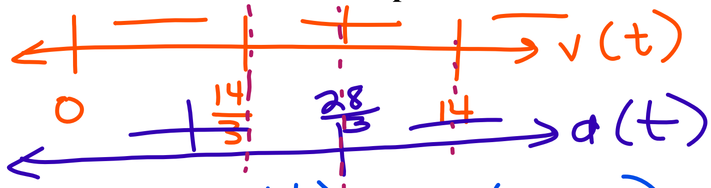
$y(-\frac{\pi}{6}) = -\sec(-\frac{\pi}{6}) = \frac{-1}{\cos(-\frac{\pi}{6})} = \frac{-1}{\frac{\sqrt{3}}{2}} = \frac{-2}{\sqrt{3}}$

$\frac{dy}{dx} = -\sec x \tan x$ $\quad \left.\frac{dy}{dx}\right|_{x=-\frac{\pi}{6}} = -\sec(-\frac{\pi}{6})\tan(-\frac{\pi}{6}) = \frac{-2}{\sqrt{3}} \cdot \frac{-1}{\sqrt{3}} = \frac{2}{3}$

Tangent slope $= \frac{2}{3}$, therefore NORMAL $= -\frac{3}{2}$ $\quad \boxed{y + \frac{2}{\sqrt{3}} = -\frac{3}{2}\left(x + \frac{\pi}{6}\right)}$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times t when the particle changes directions, the intervals of time when the particle is moving left and moving right, the times t when the acceleration is 0, and the intervals of time when the particle is slowing down and speeding up.

10) $s(t) = -t^3 + 28t^2 - 196t$



$v(t) = s'(t) = -3t^2 + 56t - 196$

$a(t) = s''(t) = -6t + 56$

$-3t^2 + 56t - 196 = 0$

$-(3t^2 - 56t + 196) = 0$

$-(3t - 14)(t - 14) = 0$

$t = \frac{14}{3}, 14$

LEFT $(0, \frac{14}{3})$ and $(14, \infty)$

Right $(\frac{14}{3}, 14)$

$-6t + 56 = 0$

$\frac{-6t}{-6} = \frac{-56}{-6}$

$t = \frac{28}{3}$

speeding up $(\frac{14}{3}, \frac{28}{3})$ $(14, \infty)$

slowing down $(0, \frac{14}{3})$ $(\frac{28}{3}, 14)$

You are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

Helpful

$f(g(x))$

$f(x) \to f'(x)$

$g(x) \to g'(x)$

$f'(g(x))\, g'(x)$

11)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-1	1	2
2	4	-1	3	$\frac{3}{2}$
3	3	$-\frac{3}{2}$	4	$\frac{3}{2}$
4	1	$-\frac{1}{2}$	6	0
5	2	1	4	$-\frac{3}{2}$
6	3	1	3	-1

Part 1) Given $h_1(x) = (f(x))^2$, find $h_1'(3)$

Part 2) Given $h_2(x) = f(g(x))$, find $h_2'(3)$

1) $h_1'(x) = 2\, f(x)\, f'(x)$

$h'(3) = 2\, f(3)\, f'(3)$

$= 2 \cdot 3 \cdot \frac{-3}{2} = \boxed{-9}$

2) $h_2'(x) = f'(g(x))\, g'(x)$

$h'(3) = f'(g(3))\, g'(3) = f'(4)\, \frac{3}{2}$

$= -\frac{1}{2} \cdot \frac{3}{2} = \boxed{\frac{-3}{4}}$