AP Calculus Name________________________________

Chain Rule, Deriving Trig Functions and Position Notes

Date________________

Differentiate each function with respect to the given variable.

$\frac{dh}{dt}$ or $h'(t)$

1) $h = \sqrt{\frac{5t^2 + 2}{3t^4 + 5}}$ $\rightarrow h = \left(\frac{5t^2+2}{3t^4+5}\right)^{\frac{1}{2}}$

$$\frac{dh}{dt} = \frac{1}{2}\left(\frac{5t^2+2}{3t^4+5}\right)^{-\frac{1}{2}} \cdot \left(\frac{(3t^4+5)(10t) - (5t^2+2)(12t^3)}{(3t^4+5)^2}\right)$$

2) $g = (-s^2 - 5)^{-3}\sqrt{3s^3 - 5}$

$g = (-s^2-5)^{-3} \cdot (3s^3-5)^{\frac{1}{2}}$

$$\frac{dg}{ds} = \underbrace{(-s^2-5)^{-3}}_{\text{NoDer (1)}} \cdot \underbrace{\frac{1}{2}(3s^3-5)^{-\frac{1}{2}} \cdot 9s^2}_{\text{Der (2)}} + \underbrace{(3s^3-5)^{\frac{1}{2}}}_{\text{NoDer(2)}} \cdot \underbrace{(-3)(-s^2-5)^{-4} \cdot (-2s)}_{\text{Der (1)}}$$

3) $h(s) = (4s^3 + 1)^{-3}$

$$h'(s) = -3(4s^3+1)^{-4} \cdot (12s^2) = \frac{-36s^2}{(4s^3+1)^4}$$

4) $f = \left((3x+5)^3 - 2\right)^{\frac{1}{4}}$

O $\quad t^{\frac{1}{4}} \longrightarrow \frac{1}{4}t^{-\frac{3}{4}}$

$t^3 - 2 \longrightarrow 3t^2$

I $\quad 3x+5 \longrightarrow 3$

$$f'(x) = \frac{1}{4}\left((3x+5)^3 - 2\right)^{-\frac{3}{4}} \cdot 3(3x+5)^2 \cdot 3$$

Differentiate each function with respect to *x*.

5) $f(x) = \dfrac{\csc 4x^4}{\cot x^3}$

$$f'(x) = \frac{\cot(x^3)\left(-\csc(4x^4)\cot(4x^4)\cdot 16x^3\right) - \csc(4x^4)\left(-\csc^2(x^3)\,3x^2\right)}{\left(\cot(x^3)\right)^2}$$

$\cot^2(x^3)$

same statement

6) $f(x) = \cot\left(\sin 3x^3\right)$

O $\quad \cot t \longrightarrow -\csc^2 t$

$\sin t \longrightarrow \cos t$

I. $\quad 3x^3 \longrightarrow 9x^2$

$$f'(x) = -\csc^2\left(\sin(3x^3)\right)\cdot \cos\left(3x^3\right)\cdot 9x^2$$

7) $y = + \frac{-16x}{(2x-5)^2}$

$$\frac{dy}{dx} = \frac{(2x-5)^{\cancel{2}}(-16) - (-16x)\,2\,\cancel{(2x-5)}^{1}\,2}{(2x-5)^{\cancel{4}\,3}}$$

$$\Rightarrow -32x + 80 + 64x$$

$$\Rightarrow \boxed{\frac{32x+80}{(2x-5)^3}} \quad \text{or} \quad \boxed{\frac{16(2x+5)}{(2x-5)^3}}$$

For each problem, find the equation of the line tangent to the function at the given point.

$\checkmark x_1, y_1 \quad \frac{dy}{dx} \checkmark$

8) $y = -\frac{16x}{(2x-5)^2}$ at $x = 2$

$$y(2) = \frac{-16(2)}{(2(2)-5)^2} = \boxed{\frac{-32}{1}}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{16(2(2)+5)}{(2(2)-5)^3} = \boxed{\frac{144}{-1}}$$

$$\boxed{y + 32 = -144(x-2)}$$

For each problem, find the equation of the line normal to the function at the given point.

9) $y = -\sec(x)$ at $x = -\frac{\pi}{6}$

$$y\left(-\frac{\pi}{6}\right) = -\sec\left(-\frac{\pi}{6}\right) = \boxed{-\frac{2}{\sqrt{3}}}$$

$$\frac{dy}{dx} = -\sec(x)\tan(x)$$

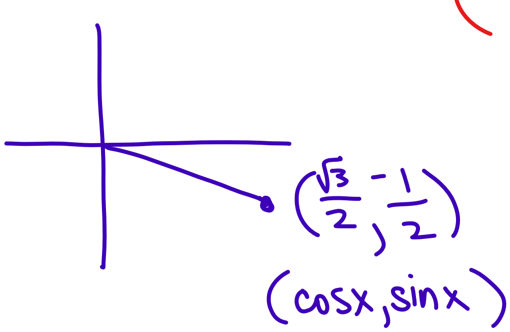
$$\left.\frac{dy}{dx}\right|_{x=-\frac{\pi}{6}} = -\sec\left(-\frac{\pi}{6}\right)\tan\left(-\frac{\pi}{6}\right)$$

$$= -\left(\frac{2}{\sqrt{3}}\right)\left(\frac{-1/2}{\sqrt{3}/2}\right)$$

$$= -\left(\frac{2}{\sqrt{3}}\right)\left(\frac{-1}{\sqrt{3}}\right)$$

$$= \frac{2}{3} \perp -\frac{3}{2}$$

$$\boxed{y + \frac{2}{\sqrt{3}} = \frac{-3}{2}\left(x + \frac{\pi}{6}\right)}$$



$a(t) = 0 = -6t + 56 \rightarrow t = \frac{56}{6} \rightarrow \frac{28}{3}$

$+$ | $-$ $a(t)$; 0, $28/3$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times t when the particle changes directions, the intervals of time when the particle is moving left and moving right, the times t when the acceleration is 0, and the intervals of time when the particle is slowing down and speeding up.

10) $s(t) = -t^3 + 28t^2 - 196t$

$v(t) = s'(t) = -3t^2 + 56t - 196$

$a(t) = v'(t) = s''(t) = -6t + 56$

$v(t) = 0 = -3t^2 + 56t - 196$

$= -(3t^2 - 56t + 196)$

$= -(3t - 14)(t - 14)$

Particle changes direction at $t = \frac{14}{3}, 14$ b/c $v(t)$ changed signs

Left $(0, \frac{14}{3}), (14, \infty)$ $v(t) < 0$

Right: $(\frac{14}{3}, 14)$ $v(t) > 0$

Particle is stopped at $t = \frac{14}{3}$ and $t = 14$

$-$ | $+$ | $-$ $v(t)$; 0, $14/3$ (0), 14 (0)

Slowing Down: $(0, \frac{14}{3}), (\frac{28}{3}, 14)$
$v(t)$ & $a(t)$ have different signs

Speeding up $(\frac{14}{3}, \frac{28}{3}), (14, \infty)$
$v(t)$ & $a(t)$ have same sign

You are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

11)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-1	1	2
2	4	-1	3	$\frac{3}{2}$
3	3	$-\frac{3}{2}$	4	$\frac{3}{2}$
4	1	$-\frac{1}{2}$	6	0
5	2	1	4	$-\frac{3}{2}$
6	3	1	3	-1

Part 1) Given $h_1(x) = (f(x))^2$, find $h_1'(3)$

Part 2) Given $h_2(x) = f(g(x))$, find $h_2'(3)$

1) $h_1'(x) = 2\,f(x)\,f'(x)$

$h_1'(3) = 2\,f(3)\,f'(3)$

$= 2 \cdot 3 \cdot -\frac{3}{2} = \boxed{-9}$

2.) $h_2'(x) = f'(g(x)) \cdot g'(x)$

$h_2'(3) = f'(g(3)) \cdot g'(3)$

$= f'(4) \cdot g'(3) \longrightarrow (-\frac{1}{2})(\frac{3}{2}) = \boxed{\frac{-3}{4}}$

