AB Calculus The Chain Rule Part 1 Homework Name: Key

1. Find dy/dx.

a) $y = (2x-7)^3$

$3(2x-7)^2(2)$

$= 6(2x-7)^2$

b) $f(x) = \cos(7x)$

$-\sin(7x)7$

$-7\sin 7x$

c) $y = x^3 \tan x$

$3x^2\tan x + x^3\sec^2 x$

d) $y = \dfrac{\tan x}{\csc x}$ $\dfrac{\csc x \sec^2 x + \tan x \cot x \csc x}{\csc^2 x}$

$= \dfrac{\sec^2 x + 1}{\csc x}$

e) $y = 3x + \sqrt{x^2+1}$ $(x^2+1)^{\frac{1}{2}}$

$3 + \frac{1}{2\sqrt{x^2+1}}(2x)$

$= 3 + \dfrac{x}{\sqrt{x^2+1}}$

f) $y = \tan(\cos x)$

$\tan x \quad \sec^2 x$

$\cos x \quad -\sin x$

$-\sec^2(\cos x)\sin x$

2. If $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, $h'(5) = -2$, find $f'(5)$ for each of the following.

a) $f(x) = 3g(x) + 2h(x)$

$3g'(x) + 2h'(x)$

$3(6) + 2(-2) = 18 - 4 = 14$

b) $f(x) = g(x)h(x)$

$g(x)h'(x) + g'(x)h(x)$

$(-3)(-2) + 6(3) = 6 + 18 = 24$

c) $f(x) = \dfrac{g(x)}{h(x)}$

$\dfrac{h(x)g'(x) - g(x)h'(x)}{h^2(x)} = \dfrac{18-6}{9} = \dfrac{12}{9} = \dfrac{4}{3}$

d) $f(x) = 4 + g(x) - 5h(x)$

$g'(x) - 5h'(x)$

$6 - 5(-2) = 6 + 10 = 16$

3. Find an equation of the line tangent to the graph of $y = \sin^2(x)$ at $x = \frac{\pi}{4}$.

$y = \sin^2(\pi/4) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

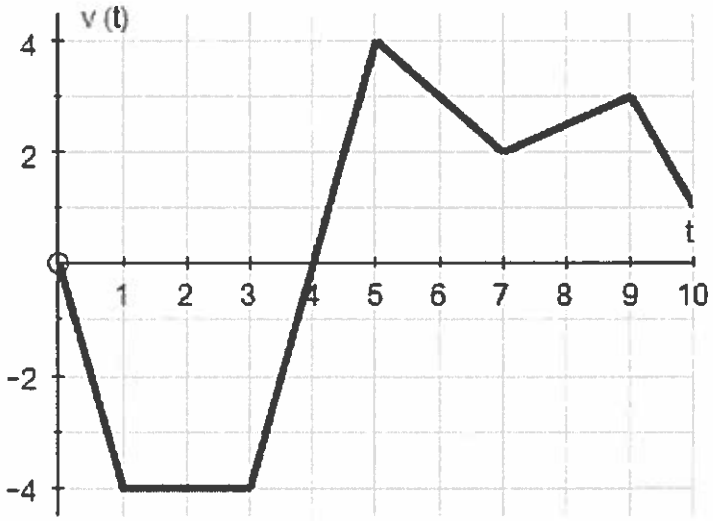
$y = (\sin x)^2$ $y' = 2\sin x \cos x$

$= 2\sin\frac{\pi}{4}\cos\frac{\pi}{4}$

$= 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{2\cdot 2}{4} = 1$

$y - \frac{1}{2} = 1\left(x - \frac{\pi}{4}\right)$

4. The graph below shows the velocity of a particle moving along the x-axis. Answer the following questions and justify each response.



a) When is the particle moving left?

$(0,4)$

b) When is the particle moving right?

$(4,10)$

c) When is the particle stopped?

4

d) When is the particle speeding up?

$(0,1),(4,5),\cup(7,9)$

e) When does the particle change direction?

4

f) When is the particle moving the fastest?

$(1,3)$, 5

g) When is the particle slowing down?

$(3,4),(5,7),(9,10)$

h) When is the particle's acceleration negative?

$(0,1),(5,7),(9,10)$

5. Find the rate of change of the function $s(t) = \sqrt{t^2 + 2t + 8}$ at the point $(2, 4)$.

$(t^2+2t+8)^{\frac{1}{2}}$

$\frac{1}{2}(t^2+2t+8)^{-\frac{1}{2}}(2t+2)$

$\frac{1}{2}(4+4+8)^{-\frac{1}{2}}(4+2) = 3\frac{1}{\sqrt{16}} = \boxed{\frac{3}{4}}$

6. Find the equation of the normal and tangent lines to the function $y = (2x - 6)^3$ at the point where $x = 1$.

$3(2x-6)^2(2)$

$6(2-6)^2$

$6(-4)^2$

$6 \cdot 16$

$= 96$

$y = (2-6)^3$

$= -4^3$

$= -64$

T: $y+64 = 96(x-1)$

N: $y+64 = -\frac{1}{96}(x-1)$