

Combinations, Permutations and Binomial Theorem

order doesn't matter

order does matter

List all possible combinations.

1) 1, 2, 3, 4, taken three at a time

123, 234, 341, 124

$$\binom{4}{3} \rightarrow \frac{4!}{(4-3)! 3!} \rightarrow \frac{4!}{1! 3!}$$

$$\rightarrow \frac{4 \cdot 3!}{1 \cdot 3!} = 4$$

2) 4, 5, 6, 7, 8, taken two at a time

$$\binom{5}{2} = \frac{5!}{(5-2)! 2!} = \frac{5!}{3! 2!}$$

$$= \frac{5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!} \cdot 2 \cdot 1} = \frac{20}{2} = 10$$

45, 56, 67, 78, 84, 85, 86,
74, 75, 64

Evaluate each expression.

3) ${}_{14}C_9$ $\binom{14}{9}$

$$\frac{14!}{5! 9!} = 2,002$$

4) ${}_{22}C_4$ $\binom{22}{4} = \frac{22!}{18! 4!}$

$$= 7,315$$

order matters

List all possible permutations.

5) 1, 2, 3, 4, taken three at a time

$$\frac{4!}{(4-3)!} = \frac{4!}{1!} = 24$$

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 123 | 124 | 134 | 234 | 213 | 214 |
| 132 | 142 | 143 | 243 | 231 | 241 |
| 321 | 342 | 341 | 412 | 423 | 413 |
| 312 | 324 | 314 | 421 | 432 | 431 |

6) 1, 2, 3, taken two at a time

$${}^3P_2 = \frac{3!}{(3-2)!}$$

$$= 6$$

- 12, 21, 23, 32,
31, 13

Evaluate each expression.

$$7) {}_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! \\ 0! = 1$$

$$6! = 720$$

$$8) {}_8P_3 = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} \\ = 336$$

Find the number of unique permutations of the letters in each word.

$$9) \text{SECTOR} \rightarrow 6! \\ 720$$

$$10) \text{BATTLE} \\ \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{2} \\ = 360$$

$$11) \text{ONGOING} \\ \frac{7!}{2!2!2!} = 630$$

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

12) Mary and Kayla are planning trips to nine countries this year. There are 11 countries they would like to visit. They are deciding which countries to skip.

$${}_{11}C_9 \text{ or } {}_{11}C_2$$

$$\frac{11!}{2!9!} \text{ or } \frac{11!}{9!2!} = 55$$

13) Maria and Daniel are planning trips to two countries this year. There are 8 countries they would like to visit. One trip will be one week long and the other two weeks.

$${}^8P_2 = \frac{8!}{6!} \\ = \frac{8 \cdot 7 \cdot 6!}{6!} = 56$$

Expand completely.

14) $(b+4)^4$

$$\begin{array}{cccccc}
 1 & 4 & 6 & 4 & 1 & \\
 b^4 & b^3 & b^2 & b & 1 & \\
 - & 4 & (4^2) & 4^3 & 4^4 &
 \end{array}$$

$$b^4 + 16b^3 + 96b^2 + 256b + 256$$

15) $(m-2)^5$

$$\begin{array}{cccccc}
 1 & 5 & 10 & 10 & 5 & 1 \\
 m^5 & m^4 & m^3 & m^2 & m & 1 \\
 - & -2 & (-2)^2 & (-2)^3 & (-2)^4 & (-2)^5
 \end{array}$$

$$m^5 - 10m^4 + 40m^3 - 80m^2 + 80m - 32$$

16) $(3y^2 - 2x^3)^3$

$$\begin{array}{cccc}
 1 & 3 & 3 & 1 \\
 (3y^2)^3 & (3y^2)^2 & 3y^2 & 1 \\
 - & -2x^3 & (-2x^3)^2 & (-2x^3)^3
 \end{array}$$

$$27y^6 - 54y^4x^3 + 36y^2x^6 - 8x^9$$

17) $(x^2 - 3y^4)^4$

$$\begin{array}{cccc}
 1 & 4 & 6 & 4 & 1 \\
 (x^2)^4 & (x^2)^3 & (x^2)^2 & (x^2)^1 & 1 \\
 - & -3y^4 & (-3y^4)^2 & (-3y^4)^3 & (-3y^4)^4
 \end{array}$$

$$x^8 - 12x^6y^4 + 54x^4y^8 - 108x^2y^{12} + 81y^{16}$$

Find each coefficient described.

18) Coefficient of $b^5 a$ in expansion of $(b - 2a)^6$

$\binom{6}{1} \rightarrow$ pascal's #

$$\frac{6!}{5!1!} = \frac{6}{1} \cdot \frac{(-2a)^1}{b^5}$$

-12

19) Coefficient of $x^2 y^{16}$ in expansion of $(2x - y^4)^6$

$$\binom{6}{4} = \frac{6!}{2!4!} = \frac{6 \cdot 5 \cdot 4!}{2 \cdot 4!} = \frac{30}{2} = 15$$

$\frac{15}{(2x)^2 (-y^4)^4} = 2^2 \cdot 15 \cdot (-1)^4 = 60$

Find each term described.

20) 5th term in expansion of $(2x^3 - y)^4$

y^4

21) 4th term in expansion of $(3x - 1)^5$

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$\binom{5}{3}$ or $\binom{5}{2}$

$= 10$ for $\Delta \#$

$10 (3x)^2 (-1)^3$

$= -90x^2$