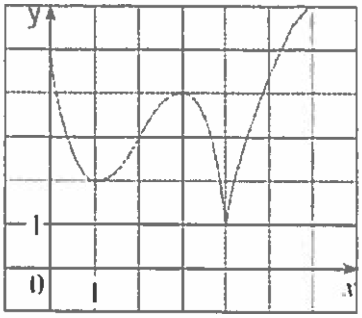
AB Calculus Concavity and the Second Derivative Test Homework Name: Key

2. Use the graph of f below to find the following.



a) The open intervals on which f is increasing.

$(1,3)$ $(4,6)$

b) The open intervals on which f is decreasing.

$(0,1)$ $(3,4)$

c) The open intervals on which f is concave downward

$(2,4)$ $(4,6)$

d) The open intervals on which f is concave upward.

$(0,2)$

e) The coordinates of the points of inflection.

$(2,3)$

2. Suppose you are given a formula for a function f.

a) How do you determine where f is increasing or decreasing?

$f'(x)=0$ or $f'(x)$ und.

If $f'(x)<0 \rightarrow f(x)$ decreasing

If $f'(x)>0 \rightarrow f(x)$ increasing

b) How do you determine where the graph of f is concave upward or downward?

$f''(x)=0$ or $f''(x)$ und

If $f''(x)>0 \rightarrow f(x)$ CCU

If $f''(x)<0 \rightarrow f(x)$ CCD

c) How do you locate inflection points?

If $f''(x)$ changes signs

3. If $f(x) = 2x^3 + 3x^2 - 36x$, find (a) the intervals on which f is increasing or decreasing, (b) the local maximum and minimum values, and (c) the intervals of concavity and inflection points.

$f'(x) = 6x^2 + 6x - 36$

$0 = 6(x^2 + x - 6)$

$0 = 6(x+3)(x-2)$

$x = -3, 2$

+ | − | +

−3 2

Inc: $(-\infty,-3)$ $(2,\infty)$

Dec: $(-3,2)$

Local max @ $-3 = 81$ Local min @ $2 = -44$

$f''(x) = 12x + 6$

$0 = 12x + 6$

$x = -1/2$

− | +

$-1/2$

CCU $(-\frac{1}{2},\infty)$

CCD $(-\infty,-\frac{1}{2})$

P.O.I @ $x = -\frac{1}{2}$

4. If $f(x) = \frac{x^2}{x^2-1}$, find (a) the intervals on which f is increasing or decreasing, (b) the local maximum and minimum values, and (c) the intervals of concavity and inflection points.

$$f'(x) = \frac{(x^2-1)(2x) - (x^2)(2x)}{(x^2-1)^2}$$

Local max: 0 @ x = 0

Local min None

$\rightarrow 2x^3 - 2x - 2x^3$

$\rightarrow \frac{-2x}{(x^2-1)^2}$ $x=0$ $x=\pm 1$

$+ \mid + \mid - \mid -$

$-1 \quad 0 \quad 1$

Inc: $(-\infty,-1)$ $(-1,0)$

Dec: $(0,1)$ $(1,\infty)$

$$f''(x) = \frac{(x^2-1)^2(-2) - (-2x)(2)(x^2-1)(2x)}{(x^2-1)^4}$$

$\rightarrow \frac{(x^2-1)(-2) + 8x^2}{(x^2-1)^3} \rightarrow \frac{6x^2+2}{(x^2-1)^3}$ $x = \pm 1$

$+ \mid - \mid +$

$-1 \quad 1$

CCU: $(-\infty,-1)$ $(1,\infty)$

CCD: $(-1,1)$

5. If $f(x) = 6x - x^2$, find all relative extrema using the second derivative test.

$f'(x) = 6 - 2x \rightarrow 0 = 6 - 2x$

$x = 3$

$f''(x) = -2$

$- \mid -$

3

rel max @ $x=3$

(b/c $f'(3)=0$ and $f''(3)<0$)

6. If $f(x) = x + \frac{4}{x}$, find all relative extrema using the second derivative test.

$f'(x) = 1 + 4x^{-2}(-1)$

$= 1 - 4x^{-2} \rightarrow$

$1 - \frac{4}{x^2} \rightarrow \frac{x^2-4}{x^2}$

$\rightarrow \frac{(x-2)(x+2)}{x^2} \rightarrow$ CN $\pm 2, 0$

rel min @ $x=2$

rel max @ $x=-2$

$f''(x) = \frac{8}{x^3}$ $x=0$

$- \mid - \mid + \mid +$ $f''(x)$

$-2 \quad 0 \quad 2$

7. If $f(x) = \sqrt{x+2}$, show that the function f satisfies the hypothesis of the Mean Value Theorem on the interval $[2, 7]$. If it does, find each value of c in (a,b) guaranteed by the theorem.

$f(2) = \sqrt{4} = 2$

$f(7) = \sqrt{9} = 3$

f(x) is cont on [2,7] and diff. on (2,7)

$\frac{3-2}{7-2} = \frac{1}{5}$

$f'(x) = \frac{1}{2\sqrt{x+2}}$

$\frac{1}{2\sqrt{x+2}} = \frac{1}{5}$

$2\sqrt{x+2} = 5$

$4(x+2) = 25$

$x+2 = \frac{25}{4}$

$\swarrow 2$

$x = \frac{25}{4} - \frac{8}{4}$

$x = \frac{17}{4} \rightarrow$ yes betw 2 & 7

so $\boxed{c = \frac{17}{4}}$

5. Find the absolute extrema of the function $f(x) = \sqrt[3]{x}(8-x)$ over the interval $[0, 8]$.

$f(x) = 8x^{\frac{1}{3}} - x^{\frac{4}{3}}$

$f'(x) = \frac{8}{3\sqrt[3]{x^2}} - \frac{4\sqrt[3]{x}}{3} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$= \frac{8-4x}{3\sqrt[3]{x^2}}$

CN: $x = 0, 2$

max

$[\ + \mid - \ ]$

$0 \quad 2 \quad 8$

check 0 & 8

$\downarrow \quad \downarrow$

$0 \quad 0$

abs max = $6\sqrt[3]{2}$ @ $x=2$

abs min = 0 @ $x=0$