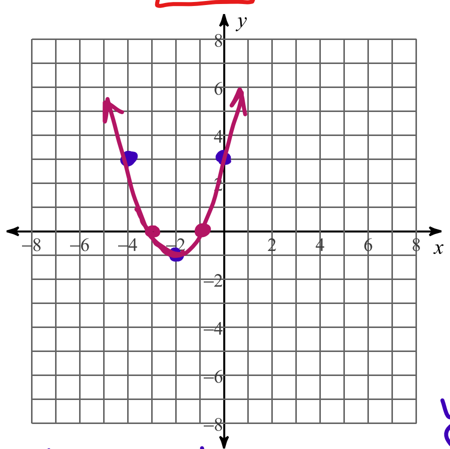
PreCalculus Name________________

Conic Sections - Circles, Parabolas and Ellipses CW 2/9/17

Date________________ Period____

Identify the vertex, focus, directrix, direction of opening, min/max value, y-intercept, and x-intercepts of each. Then sketch the graph.

1) $-x^2 - 4x + y - 3 = 0$



$y - 3 = x^2 + 4x + 4$
$+4$

$$\boxed{y + 1 = (x+2)^2 \quad \text{or} \quad y = (x+2)^2 - 1}$$

V $(-2, -1)$

Opening up

Minimum is -1

y-int $(0, 3)$

$\sqrt{0+1} = \sqrt{(x+2)^2}$

$\pm 1 = x + 2$

$x = -2 \pm 1 \rightarrow (-1, 0)\ (-3, 0)$

$p = \frac{1}{4a} \rightarrow \frac{1}{4(1)}$

focus $\left(-2, -1 + \frac{1}{4}\right)$

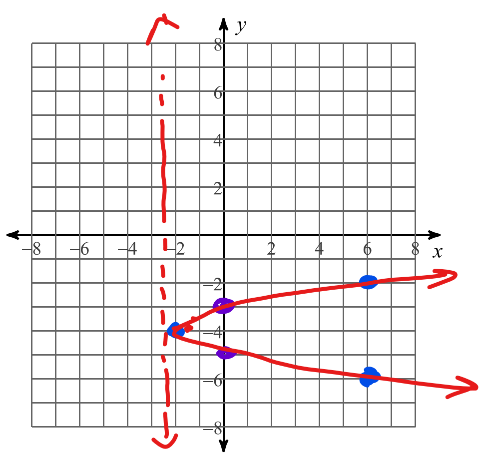
$\left(-2, -\frac{3}{4}\right)$

Directrix $y = -1 - \frac{1}{4}$

$y = -\frac{5}{4}$

Identify the vertex, focus, directrix, direction of opening, min/max value, x-intercept, and y-intercepts of each. Then sketch the graph.

2) $-2y^2 + x - 16y - 30 = 0$



$x - 30 = 2y^2 + 16y + 32$
$+32$

$= 2(y^2 + 8y + 16)$

$$\boxed{x + 2 = 2(y+4)^2}$$

vertex: $(-2, -4)$

x-int. $(30, 0)$

y-int $(0, -3)$
$(0, -5)$

$\frac{0+2}{2} = \frac{2}{2}(y+4)^2$

$\sqrt{1} = \sqrt{(y+4)^2}$

$\pm 1 = y + 4 \rightarrow y = -4 \pm 1$

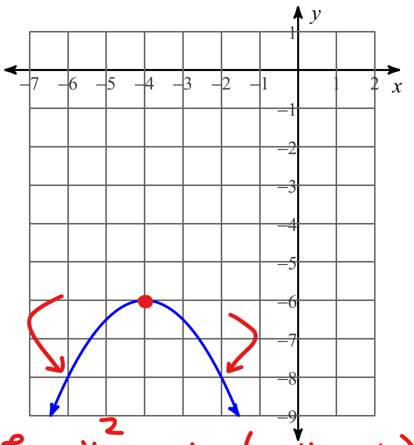
$p = \frac{1}{4(2)} = \frac{1}{8}$

F: $\left(-1\frac{7}{8}, -4\right)$

Directrix $x = -2\frac{1}{8}$

Use the information provided to write the vertex form equation of each parabola.

3)



a is negative, x^2, v $(-4,-6)$

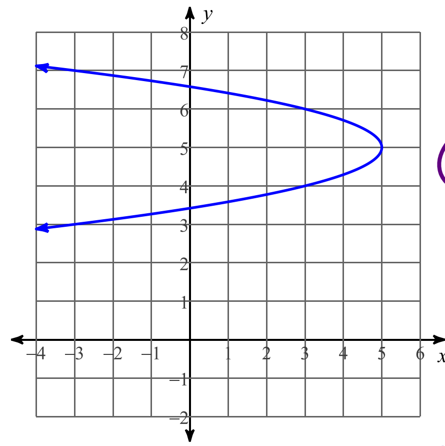
$y+6=a(x+4)^2$

$-8+6=a(-2+4)^2$

$-2=a(4) \rightarrow a=-\frac{1}{2}$

$\boxed{y+6=-\frac{1}{2}(x+4)^2}$

4)



$a<0$

$(5,5)$

y^2

$x-5=a(y-5)^2$

$3-5=a(4-5)^2$

$-2=a$

$\boxed{x-5=-2(y-5)^2}$

Use the information provided to write the standard form equation of each circle.

5) $6x=-144-24y-x^2-y^2$
Translated 2 left, 5 down

$x^2+6x+\underline{\ 9\ }+y^2+24y+\underline{\ 144\ }=-144+144+9$

$(x+3)^2+(y+12)^2=9$

Translating by 2 left, 5 down $\rightarrow$ $\boxed{(x+5)^2+(y+17)^2=9}$

$(x-h)^2+(y-k)^2=r^2$

6) Three points on the circle:
(–5, 18), (3, –12), and (–16, –1)

$(-5-h)^2+(18-k)^2=r^2 \rightarrow 25+10h+h^2+324-36k+k^2=r^2$

$(3-h)^2+(-12-k)^2=r^2 \rightarrow 9-6h+h^2+144+24k+k^2=r^2$

$(-16-h)^2+(-1-k)^2=r^2 \rightarrow 256+32h+h^2+1+2k+k^2=r^2$

eq 1 – eq 2 $\rightarrow 16+16h+180-60k=0 \rightarrow 16h-60k=-196$

eq 3 – eq 2 $\rightarrow 247+38h-143-22k=0 \rightarrow 38h-22k=-104$

$19(8h-30k=-98) \rightarrow 152h-570k=-1862$

$-8(19h-11k=-52) \rightarrow -152h+88k=416$

$-482k=-1446$

$\frac{-482k}{-482}=\frac{-1446}{-482}$

$k=3$

$8h-30(3)=-98$

$8h-90=-98$

$8h=-8$

$\boxed{h=-1}$

$(3+1)^2+(-12-3)^2=r^2$

$16+225=r^2$

$\boxed{241=r^2}$

$$\boxed{(x+1)^2+(y-3)^2=241}$$

Use the information provided to write the standard form equation of each ellipse.

7) $60x + 14y = -79 - 6x^2 - y^2$

$6x^2 + 60x + \underline{150} + y^2 + 14y + \underline{49} = -79 + 49 + 150$

$6(x^2 + 10x \underline{+25})$

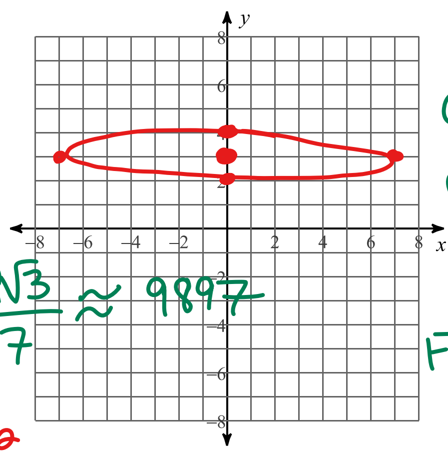
$6(x+5)^2 + (y+7)^2 = 120$

$$\boxed{\frac{(x+5)^2}{20} + \frac{(y+7)^2}{120} = 1}$$

Identify the center, vertices, co-vertices, foci, and eccentricity of each. Then sketch the graph.

8) $49y^2 = -392 + 294y - x^2$

$e = \frac{c}{a}$



$c^2 = a^2 - b^2$

$c^2 = 49 - 1$

$c^2 = 48$

$c = \sqrt{48} = 4\sqrt{3}$

Foci $(\pm 4\sqrt{3}, 3)$

$e = \frac{4\sqrt{3}}{7} \approx .9897$

$x^2 + 49y^2 - 294y \underline{+441} = -392 + 441$

$+49(y^2 - 6y \underline{+9})$

$\frac{x^2}{49} + \frac{49(y-3)^2}{49} = \frac{49}{49}$

$$\boxed{\frac{x^2}{49} + (y-3)^2 = 1}$$

C: $(0,3)$

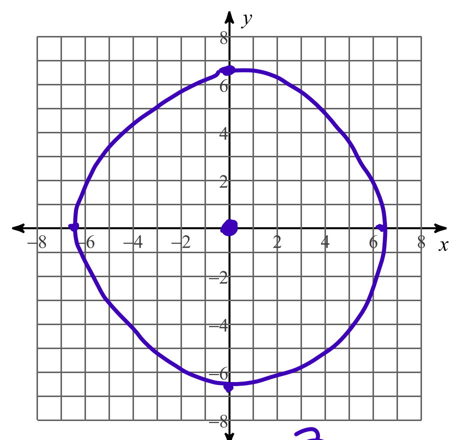
vertices: $(-7,3)$ $(7,3)$

co-vertices $(0,4)$ $(0,2)$

9) $0 = 360 - 8y^2 - 9x^2$

$e = \frac{\sqrt{5}}{3\sqrt{5}}$

$e = \frac{1}{3}$



$9x^2 + 8y^2 = 360$

$$\boxed{\frac{x^2}{40} + \frac{y^2}{45} = 1}$$

C: $(0,0)$

$a^2 = 45 \rightarrow a = \sqrt{45} = 3\sqrt{5}$

$b^2 = 40 \rightarrow b = \sqrt{40} = 2\sqrt{10}$

$c^2 = a^2 - b^2 = 5 \rightarrow c = \sqrt{5}$

Foci. $(0, \pm\sqrt{5})$

c applied to x so a^2 on bottom of x

Use the information provided to write the standard form equation of each ellipse.

10) Foci: $(-10 + 3\sqrt{17}, 9)$, $(-10 - 3\sqrt{17}, 9)$

Point on the ellipse: $\left(\frac{\sqrt{507} - 20}{2}, 7\right)$

c $(-10, 9)$ $c = 3\sqrt{17} \rightarrow c^2 = 153$

$a^2 - b^2 = 153$

$$\frac{(x+10)^2}{a^2} + \frac{(y-9)^2}{a^2-153} = 1$$

$$\frac{\left(\frac{\sqrt{507}}{2}\right)^2}{a^2} + \frac{(7-9)^2}{a^2-153} = 1$$

$$\boxed{\frac{(x+10)^2}{169} + \frac{(y-9)^2}{16} = 1}$$

$$\frac{507}{4a^2} + \frac{4}{a^2-153} = 1$$

$$507(a^2-153) + 4(4a^2) = 4a^2(a^2-153)$$

$$507a^2 - 77{,}571 + 16a^2 = 4a^4 - 612a^2$$

$$0 = 4a^4 - 1{,}135a^2 + 77{,}571$$

$a \cdot c$
$= 4 \cdot 77{,}571$
$= 310{,}284$
Factor this

$$0 = 4a^4 - 676a^2 - 459a^2 + 77{,}571$$

$$= 4a^2(a^2-169) - 459(a^2-169)$$

$a^2 \neq \frac{459}{4}$
too small

$$0 = (4a^2-459)(a^2-169)$$

$$\boxed{a^2 = 169}$$