

Conic Sections- Circles, Ellipses and Parabolas

Identify the vertex, focus, directrix, direction of opening, min/max value, y-intercept, and x-intercepts of each. Then sketch the graph.

1)  $y + 1 = -2x - x^2$

$y + 1 = -(x^2 + 2x + 1)$

$y = -(x + 1)^2$

v: (-1, 0)

down

max: 0

$\frac{1}{4(-1)} = -\frac{1}{4}$

Focus:  $(-1, -\frac{1}{4})$

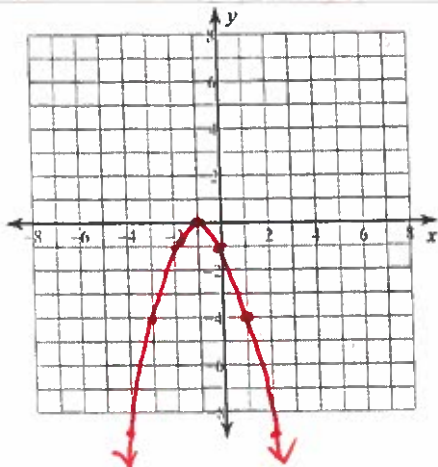
Directrix  $y = \frac{1}{4}$

$0 = -(x + 1)^2$

$x = -1$  x-int: (-1, 0)

$y = -(0 + 1)^2$

$y = -1$  y-int: (0, -1)



Identify the vertex, focus, directrix, direction of opening, x-intercept, and y-intercepts of each. Then sketch the graph.

2)  $-2y^2 + x + 12y - 16 = 0$

$x - 16 = 2y^2 - 12y + 18$

$= 2(y^2 - 6y + 9)$

$x + 2 = 2(y - 3)^2$

Right

v: (-2, 3) F:  $(-1\frac{7}{8}, 3)$

$\frac{1}{4(2)} = \frac{1}{8}$

D:  $x = -2\frac{1}{8}$

$x + 2 = 2(0 - 3)^2$   
 $x = 18 - 2$

$0 + 2 = 2(y - 3)^2$

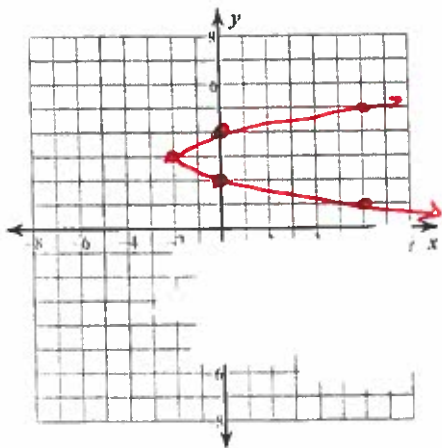
x-int: (16, 0)

$1 = (y - 3)^2$

$\pm 1 = y - 3$

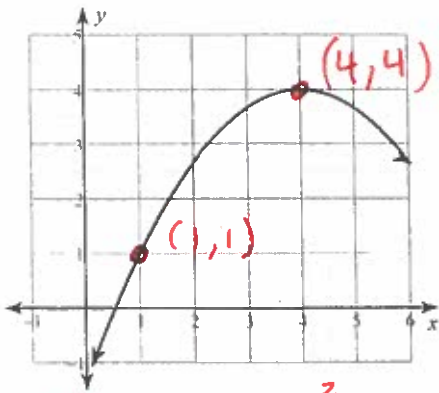
$3 \pm 1 = y$

y-int: (0, 4) (0, 2)



Use the information provided to write the vertex form equation of each parabola.

3)



$$y - k = a(x - h)^2$$

$$y - 4 = a(x - 4)^2$$

$$1 - 4 = a(1 - 4)^2$$

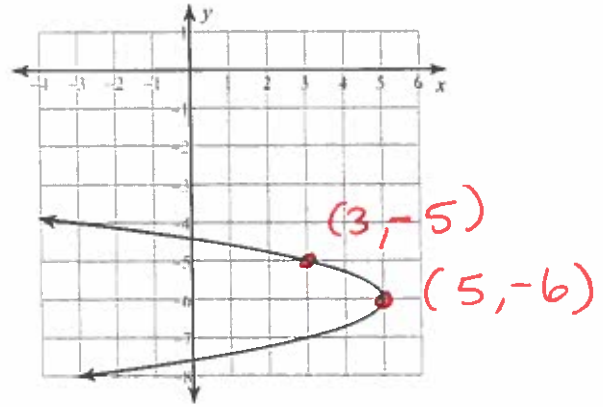
$$-3 = a(-3)^2$$

$$-3 = 9a$$

$$a = -\frac{1}{3}$$

$$y - 4 = -\frac{1}{3}(x - 4)^2$$

4)



$$x - h = a(y - k)^2$$

$$x - 5 = a(y + 6)^2$$

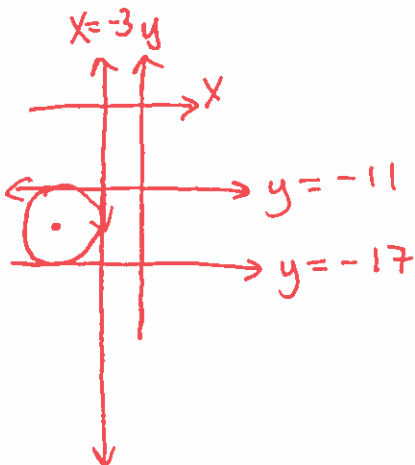
$$3 - 5 = a(-5 + 6)^2$$

$$-2 = a$$

$$x - 5 = -2(y + 6)^2$$

Use the information provided to write the standard form equation of each circle.

- 5) Center lies in the third quadrant  
Tangent to  $x = -3$ ,  $y = -17$ , and  $y = -11$



diameter = 6

so  $r = 3$

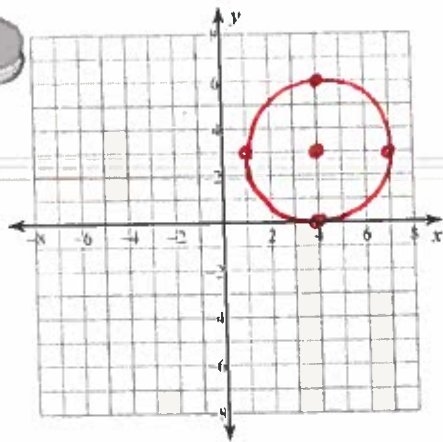
$y = -14$  of center

$x = -6$  of center

$$(x + 6)^2 + (y + 14)^2 = 9$$

Identify the center and radius of each. Then sketch the graph.

6)  $-6y + y^2 = 8x - x^2 - 16$



$$x^2 - 8x + 16 + y^2 - 6y + 9 = -16 + 25$$

$$(x - 4)^2 + (y - 3)^2 = 9$$

$C: (4, 3)$

$r = 3$

Use the information provided to write the standard form equation of each ellipse.

7) Vertices:  $(5, 6), (5, -14)$   
Foci:  $(5, 4), (5, -12)$

$\frac{6 + (-14)}{2} = -4$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$C: (5, -4)$

$$\frac{(x-5)^2}{b^2} + \frac{(y+4)^2}{a^2} = 1$$

$V: (5, 6)$  is 10 units away from center

$a = 10, a^2 = 100$

$c^2 = a^2 - b^2$

$F: (5, 4)$  is 8 units away from center

$8^2 = 100 - b^2$

$b^2 = 36$

Final:  $\frac{(x-5)^2}{36} + \frac{(y+4)^2}{100} = 1$

8)  $4y^2 + 3x^2 - 365 = 40y - 30x$

$3x^2 + 30x + 75 + 4y^2 - 40y + 100 = 365 + 75$

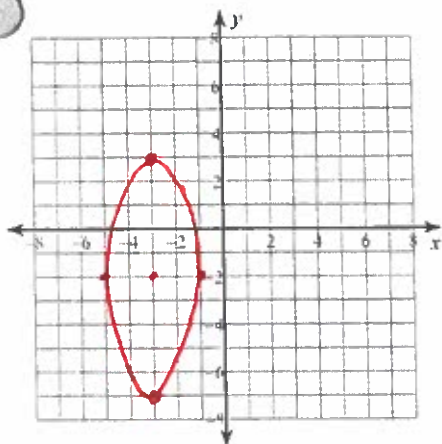
$3(x^2 + 10x + 25) + 4(y^2 - 10y + 25) = 440$

$$\frac{3(x+5)^2}{540} + \frac{4(y-5)^2}{540} = \frac{540}{540}$$

$$\frac{(x+5)^2}{180} + \frac{(y-5)^2}{135} = 1$$

Identify the center, vertices, co-vertices, foci, and eccentricity,  $\frac{c}{a}$  of each. Then sketch the graph.

9)  $150x + 25x^2 + 4y^2 + 141 = -16y$



C:  $(-3, -2)$

V:  $(-3, 3)$   $(-3, -7)$

Co-V:  $(-1, -2)$   $(-5, -2)$

$c^2 = 25 - 4 = 21$   $c = \sqrt{21}$

$$25x^2 + 150x + 225 + 4y^2 + 16y + 16 = -141 + 225 + 16$$

$$25(x^2 + 6x + 9) + 4(y^2 + 4y + 4) = 100$$

$$\frac{25(x+3)^2}{100} + \frac{4(y+2)^2}{100} = \frac{100}{100}$$

$$\frac{(x+3)^2}{4} + \frac{(y+2)^2}{25} = 1$$

Foci:  $(-3, -2 \pm \sqrt{21})$

$e = \frac{\sqrt{21}}{5}$

Use the information provided to write the vertex form equation of each parabola.

10) Focus:  $(6, \frac{15}{8})$ , Directrix:  $y = \frac{17}{8}$

horizontal  
so  $y = x^2$

$$y - k = a(x - h)^2$$

$$\frac{15}{8} + \frac{17}{8} = \frac{32}{8} = 4 \quad 4 \div 2 = 2$$

V:  $(6, 2)$

$$y - 2 = a(x - 6)^2$$

$$p = \frac{1}{4a} \text{ or } a = \frac{1}{4p} = \frac{1}{4(\frac{1}{8})} = -\frac{1}{2} = -2$$

$$y - 2 = -2(x - 6)^2$$

11) Focus:  $(\frac{71}{16}, 2)$ , Directrix:  $x = \frac{57}{16}$

vertical  
so  $x = y^2$

$$x - h = a(y - k)^2$$

$$\frac{71}{16} + \frac{57}{16} = \frac{128}{16} \rightarrow 8 \div 2 = 4$$

V:  $(4, 2)$

$$x - 4 = a(y - 2)^2$$

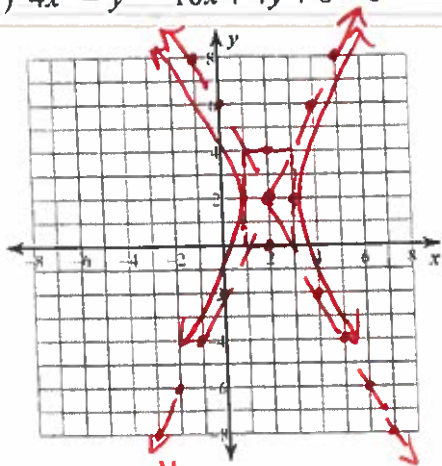
$$a = \frac{1}{4(\frac{7}{16})} = \frac{1}{7/4} = \frac{4}{7}$$

$$x - 4 = \frac{4}{7}(y - 2)^2$$

## Conic Sections- Hyperbolas and Systems

Identify the vertices, foci, and asymptotes of each. Then sketch the graph.

1)  $4x^2 - y^2 - 16x + 4y + 8 = 0$



"center" = 2, 2

$$c^2 = a^2 + b^2$$

$$= 1 + 4 = 5$$

$$c = \sqrt{5}$$

$$\text{asy: } y - 2 = \pm 2(x - 2)$$

$$v: (3, 2) (1, 2)$$

$$F: (2 \pm \sqrt{5}, 2)$$

$$4x^2 - 16x + 16 - y^2 + 4y - 4 = -8$$

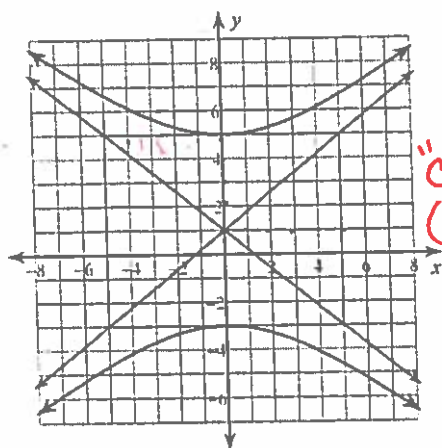
$$4(x^2 - 4x + 4) - (y^2 - 4y + 4) = -4$$

$$\frac{4(x-2)^2}{4} - \frac{(y-2)^2}{4} = \frac{4}{4}$$

$$(x-2)^2 - \frac{(y-2)^2}{4} = 1$$

Use the information provided to write the standard form equation of each hyperbola.

2)



"center"  
(0, 1)

$$-\frac{x^2}{b^2} + \frac{(y-1)^2}{a^2} = 1$$

$$-\frac{x^2}{25} + \frac{(y-1)^2}{16} = 1$$

3) Center at (-6, 2)

Transverse axis is vertical and 18 units long  
Conjugate axis is 14 units long

$$-\frac{(x+6)^2}{49} + \frac{(y-2)^2}{81} = 1$$

x-axis is transverse

4) Vertices: (9, -7), (3, -7)  
Foci: (11, -7), (1, -7)

$$\frac{(x-6)^2}{9} - \frac{(y+7)^2}{16} = 1$$

"c" = (6, -7)

9 ↔ 3 = 6 ÷ 2 → 3 → a

11 ↔ 1 → 10 ÷ 2 → 5

$c^2 = a^2 + b^2$      $5^2 = 3^2 + b^2$      $25 = 9 + b^2$

State if the point given is a solution to the system of equations.

5)  $-y^2 + x - 3y + 1 = 0$   
 $17y^2 + x + 51y - 71 = 0$   
Point: (3, 1)

$-1 + 3 - 3 + 1 = 0 ✓$   
 $17(1) + 3 + 51 - 71 = 0 ✓$

yes.

Solve each system of equations.

6)  $8x^2 - 19x + 6y - 1 = 0$   
 $x - 2y + 3 = 0$

7)  $5y^2 - 18x + 20y + 2 = 0$   
 $(20x^2 + 5y^2 - 178x + 20y - 178 = 0)$

$x = 2y - 3$

$8(2y-3)^2 - 19(2y-3) + 6y - 1 = 0$

$8(4y^2 - 12y + 9) - 38y + 57 + 6y - 1 = 0$

$32y^2 - 96y + 72 - 38y + 57 + 6y - 1 = 0$

$32y^2 - 128y + 128 = 0$

$32(y^2 - 4y + 4) = 0$

$32(y-2)^2 = 0$

$y = 2$      $x = 2(2) - 3$   
 $x = 1$

(1, 2)

ANS:

(9, -8)

(9, 4)

(-1, -2)

$-20x^2 + 160x + 180 = 0$

$-20(x^2 - 8x - 9) = 0$

$-20(x-9)(x+1) = 0$

$x = 9$      $x = -1$

$5y^2 - 18(9) + 20y + 2 = 0$

$5y^2 + 20y - 160 = 0$

$5(y^2 + 4y - 32) = 0$

$5(y+8)(y-4) = 0$

$y = -8$      $y = 4$

$5y^2 + 18 + 20y + 2 = 0$

$5y^2 + 20y + 20 = 0$

$5(y^2 + 4y + 4) = 0$

$5(y+2)^2 = 0$

$y = -2$