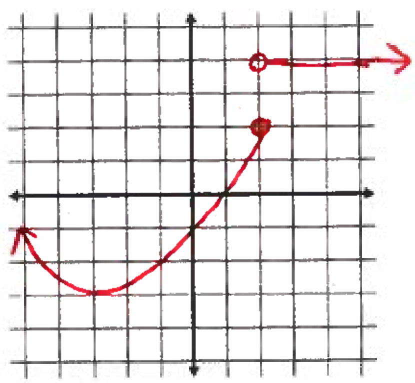
AB Calculus Continuity Part I Homework Name: Key

1. What is the definition of continuity? Hint: the answer has nothing to do with drawing.

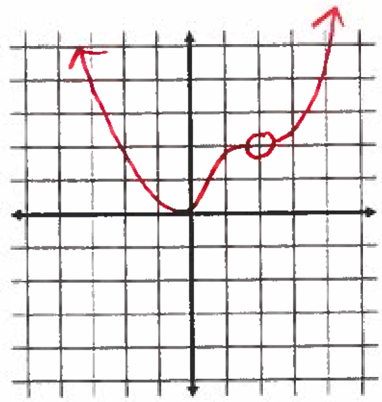
$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = \lim_{x\to c} f(x) = f(c)$

2. Sketch a possible graph for each function described.

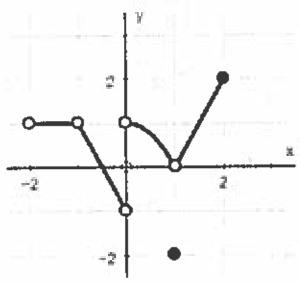
a) $f(2)$ exists, but $\lim_{x\to 2} f(x)$ does not exist.



b) $\lim_{x\to 2} f(x)$ exists, but $f(2)$ does not exist.



3. Use the function $g(x)$ defined and graphed below to answer the following questions.



$$g(x) = \begin{cases} 1, & -2 < x < -1 \\ -2x - 1, & -1 < x < 0 \\ 1 - x^2, & 0 < x < 1 \\ -2, & x = 1 \\ 2x - 2, & 1 < x \le 2 \end{cases}$$

a) Does $g(1)$ exist? yes, -2

b) Does $\lim_{x\to 1} g(x)$ exist? yes, 0

c) Does $\lim_{x\to 1} g(x) = g(1)$? NO

d) Is g continuous at $x = 1$? NO

e) Is g defined at $x = -1$? NO

f) Is g continuous at $x = -1$? NO

g) For what values of x is g continuous?

$(-2,-1) \cup (-1,0) \cup (0,1) \cup (1,2]$

h) What value should be assigned to $g(-1)$ to make the extended function continuous at $x = -1$?

1

i) What new value should be assigned to $g(1)$ to make the new function continuous at $x = 1$?

0

j) Is it possible to extend g to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

NO, b/c it's a jump which is nonremovable.

4. Let $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$. Find a value of a so that the function f is continuous. Justify your answer using the definition of continuity.

$3^2 - 1 = 2 \cdot a \cdot 3$

$8 = 6a \qquad a = \frac{4}{3}$

$\lim_{x \to 3} f(x) = 8 = f(3)$ if $a = \frac{4}{3}$

5. Determine the points of discontinuity and identify their type for each of the following functions.

a) $f(x) = \frac{1}{(x-2)^2}$

$x = 2$

V. A.

b) $f(x) = \frac{x-1}{x^2 - 4x + 3}$

$(x-1)(x-3)$

$x = 1$ Hole (removable)

$x = 3$ V.A.

c) $f(x) = \frac{|x|}{x}$

$x = 0$

Jump

6. For each of the following, write an extended function so that the given function is continuous at the indicated point.

a) $h(x) = \frac{\sin(5x)}{x} \cdot \frac{5}{5}$ at $x = 0$.

$\frac{\sin 5x}{5x} \cdot 5 = 5$

1

$h(x) = \begin{cases} \frac{\sin 5x}{x} & x \neq 0 \\ 5 & x = 0 \end{cases}$

b) $k(x) = \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$ at $x = 4$.

$\frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}} \to \sqrt{4}+2 = 2+2 = 4$

$k(x) = \begin{cases} \frac{x-4}{\sqrt{x}-2} & x \neq 4 \\ 4 & x = 4 \end{cases}$

7. Let $g(x) = \frac{x^2+5x+6}{x^2+7x+10}$. $\frac{(x+2)(x+3)}{(x+2)(x+5)}$

a) Find the domain of $g(x)$.

$x \neq -2, -5 \quad (-\infty, -5) \cup (-5, -2) \cup (-2, \infty)$

b) Find $\lim_{x \to c} g(x)$ for all values of c where $g(x)$ is not defined.

$\lim_{x \to -2} \frac{1}{3} \qquad \lim_{x \to -5}$ DNE

c) Find any horizontal asymptotes and justify your response.

H.A. $y = 1$ Degree equal top/bottom so $y = \frac{a}{b}$

d) Find any vertical asymptotes and justify your response.

$x = -5 \to$ doesn't cancel

e) Write an extended function so that $g(x)$ is continuous at $x = -2$. Use the definition of continuity to justify your response.

$g(x) = \begin{cases} \frac{x^2+5x+6}{x^2+7x+10} & x \neq -2 \\ \frac{1}{3} & x = -2 \end{cases}$

now $\lim_{x \to -2} g(x) = \frac{1}{3}$

$g(-2) = \frac{1}{3}$