AP Calculus Name_____

More Convoluted Cray U-Sub Notes

Date_____ Period____

Evaluate each indefinite integral.

1) $\int \frac{x^2-1}{x^2+1}\,dx$

$$\begin{array}{r} 1 \\ x^2+1 \overline{)\, x^2-1} \\ -(x^2+1) \\ \hline -2 \end{array}$$

$$= \int \left(1 - \frac{2}{x^2+1}\right) dx$$

$$= x - 2\tan^{-1} x + C$$

2) $\int (\sin x + \cos x)^2\,dx$

$$\int (\sin^2 x + 2\cos x \sin x + \cos^2 x)\,dx$$

$$= \int (1 + 2\cos x \sin x)\,dx$$

$$= \int 1\,dx + 2\int \cos x \sin x\,dx$$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-du = \sin x\,dx$

$$x - 2\int u\,du$$

$$x - \frac{2u^2}{2} + C$$

$$= x - \cos^2 x + C$$

3) $\int \frac{2}{x^2-6x+10}\,dx$

Complete the square

$x^2 - 6x \underline{+9} + 10 - 9$

$(x-3)^2 + 1$

$$= \int \frac{2}{(x-3)^2+1}\,dx$$

Just like #1, this is in the form of $\tan^{-1} x$

$$= 2\tan^{-1}(x-3) + C$$

4) $\int \frac{3x+2}{\sqrt{1-x^2}}\,dx$

$\int \left(\frac{3x}{\sqrt{1-x^2}} + \frac{2}{\sqrt{1-x^2}}\right) dx$

$u = 1-x^2$

$\frac{du}{dx} = -2x$

$-\frac{1}{2}du = x\,dx$

$\frac{-3}{2}\int \frac{1}{\sqrt{u}}\,du$

$-\frac{3}{2}(2)u^{\frac{1}{2}}$

$= -3(1-x^2)^{\frac{1}{2}} + 2\sin^{-1}x + C$

5) $\int \frac{x^5 - 35x}{x^2+6}\,dx$

$$\begin{array}{r} x^3 - 6x \\ x^2+6 \overline{)\, x^5 - 35x} \\ -(x^5 + 6x^3) \\ \hline -6x^3 - 35x \\ -(-6x^3 - 36x) \\ \hline x \end{array}$$

$\int \left(x^3 - 6x + \frac{x}{x^2+6}\right) dx$

$u = x^2+6$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + C$

$= \frac{x^4}{4} - \frac{6x^2}{2} + \frac{1}{2}\ln|x^2+6| + C$

6) $\int \frac{e^x + 4}{e^x}\,dx$

$= \int \left(\frac{e^x}{e^x} + \frac{4}{e^x}\right) dx$

$= \int (1 + 4e^{-x})\,dx$

$= x - 4e^{-x} + C$

7) $\int \frac{x+1}{x-1}\,dx$

$$x-1 \overline{)\, x+1}$$

$$1$$

$$-(x-1)$$

$$2$$

$$\int \left(1 + \frac{2}{x-1}\right) dx$$

$$= x + 2\ln|x-1| + C$$

8) $\int \frac{1}{x^2+10x+26}\,dx$

$$\int \frac{1}{x^2+10x\underline{+25}+26\underline{-25}}\,dx$$

$$\int \frac{1}{(x+5)^2+1}\,dx$$

$$= \tan^{-1}(x+5) + C$$

9) $\int \cot x\,dx$

$$= \int \frac{\cos x}{\sin x}\,dx$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \ln|\sin x| + C$$

10) $\int \tan^2 x\,dx$

$$= \int (\sec^2 x - 1)\,dx$$

$$= \tan x - x + C$$

11) $\int \frac{1}{\sqrt{4-x^2}}\,dx$

$$\int \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}\,dx$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx$$

$u = \frac{1}{2}x \rightarrow \frac{du}{dx} = \frac{1}{2} \rightarrow 2du = dx$

$$2\,\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$\sin^{-1}u + C \rightarrow \boxed{\sin^{-1}\left(\frac{x}{2}\right) + C}$

12) $\int \frac{3}{16+x^2}\,dx$

$$\int \frac{3}{16\left(1+\frac{x^2}{16}\right)}\,dx$$

$$\frac{3}{16}\int \frac{1}{1+\left(\frac{x}{4}\right)^2}\,dx$$

$u = \frac{x}{4} \rightarrow \frac{du}{dx} = \frac{1}{4} \rightarrow 4du = dx$

$$4\,\frac{3}{16}\int \frac{1}{1+u^2}\,du$$

$\frac{3}{4}\tan^{-1}u + C = \boxed{\frac{3}{4}\tan^{-1}\left(\frac{x}{4}\right) + C}$

13) $\int \frac{1}{9+25x^2}\,dx$

$\int \frac{1}{9\left(1+\frac{25x^2}{9}\right)}\,dx$

$= \frac{1}{9}\int \frac{1}{1+\left(\frac{5x}{3}\right)^2}\,dx$

(we can do u-sub mentally since it's a linear transformation

$= \frac{3}{5}\cdot\frac{1}{9}\tan^{-1}\left(\frac{5x}{3}\right) + C$

14) $\int (x+1)\sqrt{2-x}\,dx$

$u = 2-x \rightarrow x = 2-u$

$\frac{du}{dx} = -1$

$-du = dx$

$-\int (2-u+1)\sqrt{u}\,du$

$-\int (3-u)\sqrt{u}\,du$

$-\int (3\sqrt{u} - u\sqrt{u})\,du$

$-\int (3u^{\frac{1}{2}} - u^{\frac{3}{2}})\,du$

$-\frac{2}{3}(3)u^{\frac{3}{2}} + \frac{2}{5}u^{\frac{5}{2}} + C$

$= -2(2-x)^{3/2} + \frac{2}{5}(2-x)^{5/2} + C$

15) $\int \frac{2x+1}{\sqrt{x+4}}\,dx$

$\int \frac{2x}{\sqrt{x+4}}dx + \int \frac{1}{\sqrt{x+4}}\,dx$

$u = x+4 \rightarrow u-4 = x$

$\frac{du}{dx} = 1 \rightarrow du = dx$

$2\int \frac{u-4}{\sqrt{u}}\,du$

$2\int (u^{\frac{1}{2}} - 4u^{-\frac{1}{2}})\,du$

$2\cdot\frac{2}{3}u^{\frac{3}{2}} - 8\,2u^{\frac{1}{2}}$

$= \frac{4}{3}(x+4)^{3/2} - 16(x+4)^{\frac{1}{2}} + 2(x+4)^{\frac{1}{2}} + C$

16) $\int \frac{1}{x\sqrt{4x^2-9}}\,dx$

$\int \frac{1}{x\sqrt{9(\frac{4x^2}{9}-1)}}\,dx$

$\int \frac{1}{3x\sqrt{(\frac{2x}{3})^2-1}}\,dx$

$= \frac{1}{3}\int \frac{1}{x\sqrt{(\frac{2x}{3})^2-1}}\,dx$

$x = \frac{3}{2}u \rightarrow u = \frac{2x}{3} \rightarrow \frac{du}{dx} = \frac{2}{3}$

$\frac{3}{2}du = dx$

$\frac{3}{2}\,\frac{1}{3}\int \frac{1}{\frac{3}{2}u\sqrt{u^2-1}}\,du$

$= \frac{2}{3}\,\frac{3}{2}\,\frac{1}{3}\int \frac{1}{u\sqrt{u^2-1}}\,du$

$= \frac{1}{3}\sec^{-1}(u) + C = \frac{1}{3}\sec^{-1}\left(\frac{2x}{3}\right) + C$

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1) $\int \frac{x^2 - 1}{x^2 + 1}\,dx$

$x - 2\tan^{-1} x + C$

2) $\int (\sin x + \cos x)^2\,dx$

$x + \sin^2 x + C$

3) $\int \frac{2}{x^2 - 6x + 10}\,dx$

$2\tan^{-1}(x - 3) + C$

4) $\int \frac{3x+2}{\sqrt{1-x^2}}\,dx$

$-3\sqrt{1-x^2} + 2\sin^{-1} x + C$

5) $\int \frac{x^5 - 35x}{x^2+6}\,dx$ $\frac{1}{4}x^4 - 3x^2 + \frac{1}{2} \cdot \ln\left|x^2 - 16\right| + C$

6) $\int \frac{e^x + 4}{e^x}\,dx$

$x - 4e^{-x} + C$

7) $\int \frac{x+1}{x-1}\,dx$

$x + 2\ln\left|x-1\right| + C$

8) $\int \frac{1}{x^2+10x+26}\,dx$

$\tan^{-1}(x+5) + C$

9) $\int \cot x\,dx$

$\ln\left|\sin x\right| + C$

10) $\int \tan^2 x\,dx$

$\tan x - x + C$

11) $\int \frac{1}{\sqrt{4-x^2}}\,dx$ $\sin^{-1}\frac{x}{2}+C$

12) $\int \frac{3}{16+x^2}\,dx$

$\frac{3}{4}\cdot\tan^{-1}\frac{x}{4}+C$

13) $\int \frac{1}{9+25x^2}\,dx$ $\frac{1}{15}\cdot\tan^{-1}\frac{5x}{3}+C$

14) $\int (x+1)\sqrt{2-x}\,dx$

$-2(2-x)^{\frac{3}{2}}+\frac{2}{5}(2-x)^{\frac{5}{2}}+C$

15) $\int \frac{2x+1}{\sqrt{x+4}}\,dx$ $\frac{4}{3}(x+4)^{\frac{3}{2}} - 14(x+4)^{\frac{1}{2}} + C$

16) $\int \frac{1}{x\sqrt{4x^2-9}}\,dx$ $\frac{1}{3} \cdot \sec^{-1}\frac{2x}{3} + C$