AP Calculus Name________________

Critical Points and Extreme Value Theorem Notes

Date________________ Period____

Determine all of the critical points for the function.

1) $g(x) = 6x^5 + 33x^4 - 30x^3 + 100$

$g'(x) = 30x^4 + 132x^3 - 90x^2$

$0 = 6x^2(5x^2 + 22x - 15)$

$0 = 6x^2(5x - 3)(x + 5)$

CP $x = -5, 0, \frac{3}{5}$

2) $f(t) = \sqrt[3]{t^2}(2t - 1)$

$f(t) = t^{\frac{2}{3}}(2t^1 - 1)$

$f(t) = 2t^{\frac{5}{3}} - t^{\frac{2}{3}}$

$f'(t) = \frac{10}{3}t^{\frac{2}{3}} - \frac{2}{3}t^{-\frac{1}{3}}$

$0 = \frac{2}{3}t^{-\frac{1}{3}}(5t - 1)$

C.P $t = 0, 1/5$ (b/c $f'(0)$ is und) $f'(\frac{1}{5}) = 0$

3) $r(w) = \dfrac{w^2 + 1}{w^2 - w - 6}$

$r'(w) = \dfrac{(w^2 - w - 6)(2w) - (w^2+1)(2w-1)}{(w^2 - w - 6)^2}$

$= \dfrac{2w^3 - 2w^2 - 12w - 2w^3 + w^2 - 2w + 1}{(w^2 - w - 6)^2}$

$= \dfrac{-w^2 - 14w + 1}{(w^2 - w - 6)^2}$

$w = \dfrac{14 \pm \sqrt{196 - 4(-1)(1)}}{2(-1)}$

$= \dfrac{14 \pm \sqrt{200}}{-2} = \dfrac{14 \pm 10\sqrt{2}}{-2} = -7 \pm 5\sqrt{2}$ CP

4) $f(x) = 6x - 4\cos 3x$

5) $h(t) = 10te^{3 - t^2}$

6) $f(x) = x^2 \ln 3x + 6$

7) $f(x) = xe^{x^2}$

For each problem, find all points of absolute minima and maxima on the given interval.

8) $y = -\frac{x^2}{2} + 3x - \frac{11}{2}$; $[1, 6]$

9) $y = -x^3 + 3x^2 - 3$; $[-1, 2]$

10) $y = -x^4 + 2x^2 + 4$; $[-2, 1]$

11) $y = \dfrac{9x}{x^2 + 9}$; $[0, 6]$

12) $y = (3x + 6)^{\frac{2}{3}}$; $[-4, -1]$

13) $y = -\cos(x)$; $[-\dfrac{\pi}{2}, \dfrac{\pi}{2}]$

For each problem, find all points of relative minima and maxima.

14) $y = x^4 - x^2 - 3$

15) $y = \dfrac{x^2}{4x - 4}$

16) $y = -(-5x + 15)^{\frac{2}{3}}$

17) $y = \sin(2x);\ [-\pi, \pi]$

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Determine all of the critical points for the function.

1) $g(x) = 6x^5 + 33x^4 - 30x^3 + 100$

$x = -5, 0, \frac{3}{5}$

2) $f(t) = \sqrt[3]{t^2}(2t - 1)$ $t = \frac{1}{5}, 0$

3) $r(w) = \frac{w^2 + 1}{w^2 - w - 6}$

$w = -7 + 5\sqrt{2}, -7 - 5\sqrt{2}$

4) $f(x) = 6x - 4\cos 3x$ $x = \frac{7\pi}{18} + \frac{2\pi}{3}n, \frac{11\pi}{18} + \frac{2\pi}{3}n$

5) $h(t) = 10te^{3 - t^2}$

$t = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

6) $f(x) = x^2 \ln 3x + 6$ $x = \frac{1}{3\sqrt{e}}$

7) $f(x) = xe^{x^2}$

No critical points

For each problem, find all points of absolute minima and maxima on the given interval.

8) $y = -\frac{x^2}{2} + 3x - \frac{11}{2}$; $[1, 6]$

Absolute minimum: $\left(6, -\frac{11}{2}\right)$
Absolute maximum: $(3, -1)$

9) $y = -x^3 + 3x^2 - 3$; $[-1, 2]$

Absolute minimum: $(0, -3)$
Absolute maxima: $(-1, 1)$, $(2, 1)$

10) $y = -x^4 + 2x^2 + 4$; $[-2, 1]$

Absolute minimum: $(-2, -4)$
Absolute maxima: $(1, 5)$, $(-1, 5)$

11) $y = \frac{9x}{x^2 + 9}$; $[0, 6]$

Absolute minimum: $(0, 0)$
Absolute maximum: $\left(3, \frac{3}{2}\right)$

12) $y = (3x + 6)^{\frac{2}{3}}$; $[-4, -1]$

Absolute minimum: $(-2, 0)$
Absolute maximum: $(-4, \sqrt[3]{36})$

13) $y = -\cos(x)$; $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Absolute minimum: $(0, -1)$
Absolute maxima: $\left(-\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$

For each problem, find all points of relative minima and maxima.

14) $y = x^4 - x^2 - 3$

Relative minima: $\left(-\frac{\sqrt{2}}{2}, -\frac{13}{4}\right), \left(\frac{\sqrt{2}}{2}, -\frac{13}{4}\right)$

Relative maximum: $(0, -3)$

15) $y = \frac{x^2}{4x - 4}$

Relative minimum: $(2, 1)$

Relative maximum: $(0, 0)$

16) $y = -(-5x + 15)^{\frac{2}{3}}$

No relative minima.

Relative maximum: $(3, 0)$

17) $y = \sin(2x)$; $[-\pi, \pi]$

Relative minima: $\left(-\frac{\pi}{4}, -1\right), \left(\frac{3\pi}{4}, -1\right)$

Relative maxima: $\left(-\frac{3\pi}{4}, 1\right), \left(\frac{\pi}{4}, 1\right)$