AB Calculus: Curve Sketching **Name:** ______________________________

In this section, we are going to combine everything we have previously learned to sketch functions without a calculator.

Guidelines for Analyzing the Graph of a Function

1. Determine the intercepts, asymptotes, and symmetry of the graph.
2. Locate the x-values for which $f'(x)$ and $f''(x)$ are either zero or undefined.
3. Use the results to determine relative extrema and points of inflection as well as intervals that the function is increasing and decreasing
4. Sketch the graph so that it supports all the information you found.

$f(x) = (x^2-4x+4)(x+1) = x^3+x^2-4x^2-4x+4x+4 = x^3-3x^2+4$

$\frac{d}{dx} \to 3x^2-6x$

$3x(x-2)$

Example 1 Analyze and sketch the graph of $f(x) = (x-2)^2(x+1)$

a) Determine the x- and y-intercepts

$(2,0)$ $(-1,0)$ $(0,4)$

b) Determine the equations of any horizontal and vertical asymptotes.

No (cubic → polynomials don't ever have asymptotes)

c) Determine the first and second derivative of the function.

$f'(x) = (x-2)^2(1) + 2(x-2)^1(1)(x+1)$

$= (x-2)(x-2+2x+2)$

$f'(x) = (x-2)(3x)$

$f''(x) = (x-2)(3)+(1)(3x)$

$= 3x-6+3x$

$= 6x-6$ or $6(x-1)$

d) Complete a sign chart for $f'(x)$ to determine the intervals where the function is increasing/decreasing.

$+ \quad - \quad +$; 0, 2 ; f'

f inc $(-\infty,0)$ $(2,\infty)$

f dec $(0,2)$

e) Determine any relative minimums or maximums.

rel max at $x=0$ $(0,4)$

rel min at $x=2$ $(2,0)$

f) Complete a sign chart for $f''(x)$ to determine the intervals where the function is concave up/down.

$- \quad +$; 1 ; f''

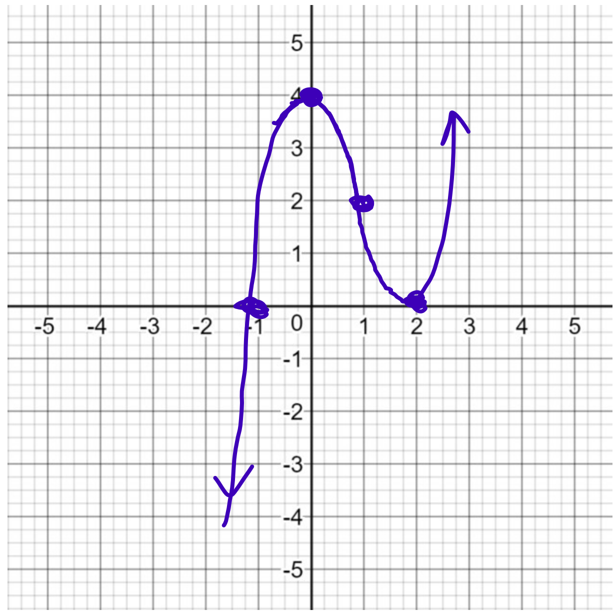
cc down $(-\infty,1)$

cc up $(1,\infty)$

g) Identify any points of inflection.

$(1,2)$

h) Complete a sketch of the function that supports all of the information above.



Example 2 Analyze and sketch the graph of $f(x) = \frac{(x^2-9)}{x^2-4}$

a) Determine the x- and y-intercepts

$(3,0)$ $(-3,0)$ $(0, \frac{9}{4})$

b) Determine the equations of any horizontal and vertical asymptotes.

V.A $x=2$ $x=-2$ HA $y=1$

c) Determine the first and second derivative of the function.

$$f'(x) = \frac{(x^2-4)(2x)-(x^2-9)(2x)}{(x^2-4)^2} \rightarrow \frac{(2x)(x^2-4-x^2+9)}{(x^2-4)^2} = \frac{\overbrace{(2x)(5)}^{10x}}{(x^2-4)^2}$$

d) Complete a sign chart for $f'(x)$ to determine the intervals where the function is increasing/decreasing.

$$f''(x) = \frac{(x^2-4)^{\cancel{2}}(10) - 10x(2\cancel{(x^2-4)})(2x)}{(x^2-4)^{\cancel{4}\,3}} \rightarrow \frac{10(x^2-4)-4x^2}{(x^2-4)^3}$$

$$= \frac{-10(+3x^2+4)}{(x^2-4)^3}$$

Dec Dec Inc Inc

e) Determine any relative minimums or maximums.

$-$ $-$ $+$ $+$ f' rel min at $x=0$ $(0, 9/4)$

-2 0 2

cc down up down

− + −

f''

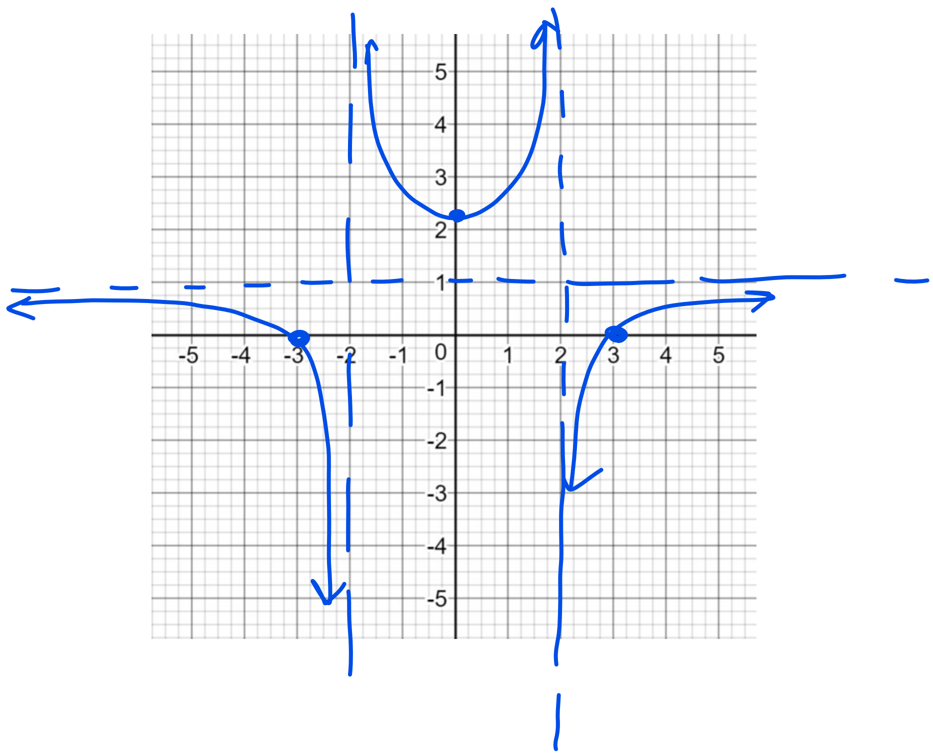
−2 2

f) Complete a sign chart for $f''(x)$ to determine the intervals where the function is concave up/down.

g) Identify any points of inflection.

NO

h) Complete a sketch of the function that supports all of the information above.



How to Draw Sections of a Curve			
Increasing Concave Up	**Increasing Concave Down**	**Decreasing Concave Up**	**Decreasing Concave Down**

Example 3 Sketch a curve illustrating a function such that

$f(-2) = 8$
$f(0) = 4$
$f(2) = 0$

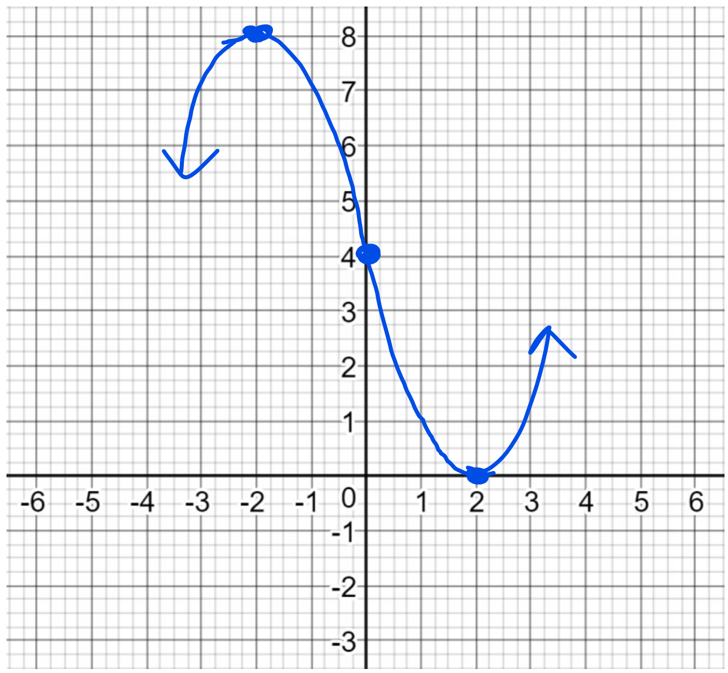
ccup right-side
$f''(x) > 0$, for $x > 0$

CP
H Tangent $\rightarrow$ $f'(2) = f'(-2) = 0$

$f'(x) < 0$ for $|x| < 2$
Dec $-2 < x < 2$

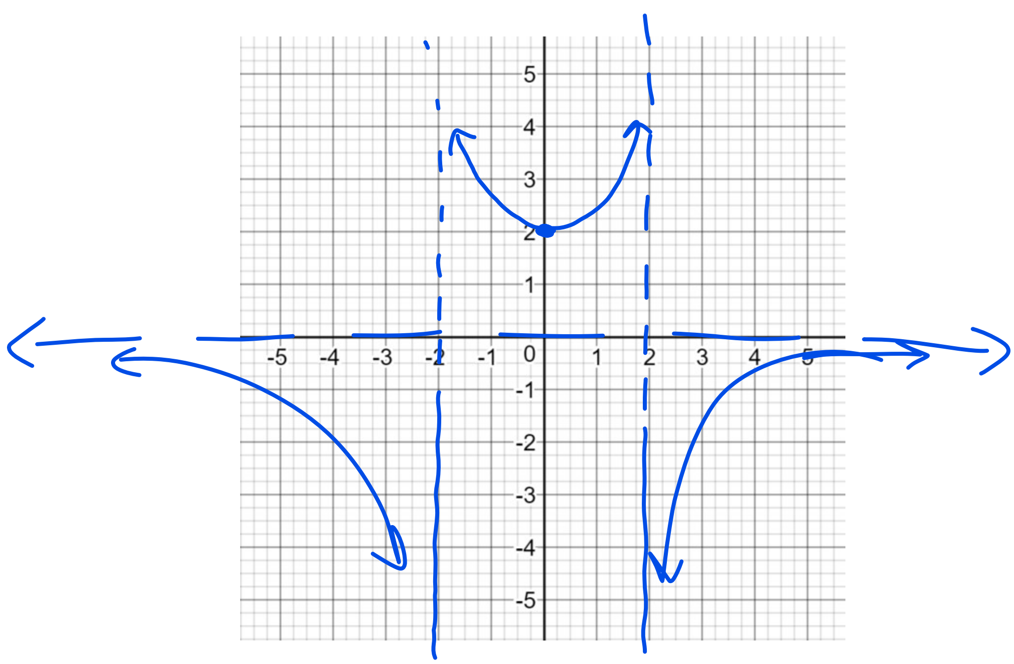
ccdown Left
$f''(x) < 0$ for $x < 0$

$f'(x) > 0$ for $|x| > 2$
Inc $x > 2$
$x < -2$



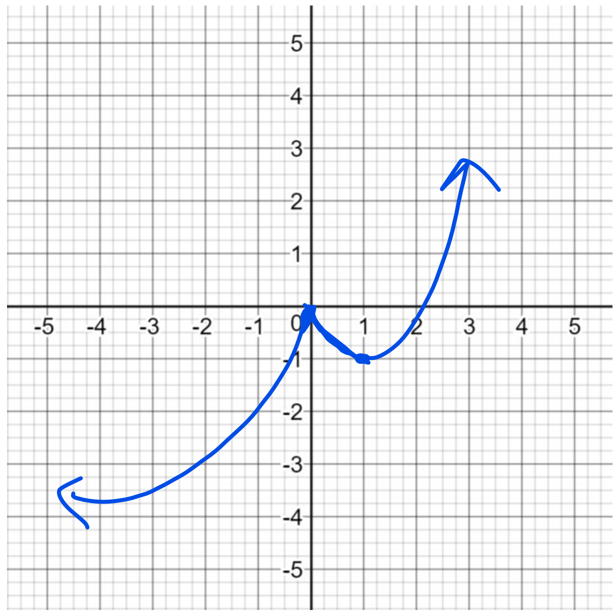
Example 4 Sketch a curve illustrating a function such that:

- It is symmetrical across the y-axis and $f(0) = 2$
- It has a horizontal asymptote: $y = 0$ and two vertical asymptotes: $x = \pm 2$
- It is increasing on $(0, 2)$ and decreasing on $(-\infty, -2)$ and $(-2, 0)$
- It is concave up on $(-2, 2)$ and concave down on $(-\infty, -2)$ and $(2, \infty)$



Example 5 Sketch a curve illustrating a function such that:

- It is increasing on $(-\infty, 0)$ and $(1, \infty)$ and decreasing on $(0, 1)$
- It has a tangent with undefined slope at the origin
- It has a horizontal tangent at $(1, -1)$
- It is concave up for all x except $x = 0$



Example 6 Let f be a function that is continuous on the interval $[0, 4]$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table below, where DNE indicates that the derivatives of f do not exist at $x = 2$. Sketch a graph of f.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive INC	0	Positive INC	DNE	Negative DEC	-3	Negative DEC
$f''(x)$	-2	Negative CCDN	0	Positive CCUP	DNE	Negative CCDN	0	Positive CCUP