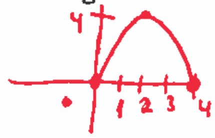
Name Key Date______________________ Period______

Worksheet 6.2—Definite Integrals & Numeric Integration

Show all work on a separate sheet of paper.



Multiple Choice

$0 = 4x - x^2$
$0 = x(4-x)$

$1(4(.5)-.5^2)+1(4(1.5)-1.5^2)+1(4(2.5)-2.5^2)$ $+1(4(3.5)-3.5^2)$

1. (Calculator Permitted) If the midpoints of 4 equal-width rectangles is used to approximate the area enclosed between the x-axis and the graph of $y = 4x - x^2$, the approximation is

   (A) 10 (B) 10.5 (C) 10.666 (D) 10.75 (E) 11

2. If $\int_2^5 f(x)dx = 18$, then $\int_2^5 (f(x)+4)dx =$ $4(3) = 12 + 18$

   (A) 20 (B) 22 (C) 23 (D) 25 (E) 30

3. $\int_{-4}^{4} (4-|x|)dx =$  $\frac{1}{2}(8)(4)$

   (A) 0 (B) 4 (C) 8 (D) 16 (E) 32

4. If $\int_a^b f(x)dx = a + 2b$, then $\int_a^b (f(x)+3)dx =$ $a + 2b + 3b - 3a = -2a + 5b$

   (A) $a+2b+3$ (B) $3b-3a$ (C) $4a-b$ (D) $5b-2a$ (E) $5b-3a$

5. The expression $\frac{1}{20}\left(\sqrt{\frac{1}{20}}+\sqrt{\frac{2}{20}}+\sqrt{\frac{3}{20}}+\ldots+\sqrt{\frac{20}{20}}\right)$ is a Riemann sum approximation for

   (A) $\int_0^1 \sqrt{\frac{x}{20}}dx$ (B) $\int_0^1 \sqrt{x}dx$ (C) $\frac{1}{20}\int_0^1 \sqrt{\frac{x}{20}}dx$ (D) $\frac{1}{20}\int_0^1 \sqrt{x}dx$ (E) $\frac{1}{20}\int_0^{20} \sqrt{x}dx$

Short Answer

6. The table below gives the values of a function obtained from an experiment. Use them to estimate $\int_0^6 f(x)dx$ using **three equal subintervals** with a) right endpoints, b) left endpoints, c) midpoints, and d) the trapezoidal rule. If the function is said to be a decreasing function, can you say whether your estimates are less than or greater than the exact value of the integral? Could any of these estimates approximate the area of the enclosed region with the x-axis? Why or why not?

x	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	−7.6	−10.5

RRAM $2(8.3)+2(2.3)+2(-10.5) = .2$ Less than

LRAM $2(9.3)+2(8.3)+2(2.3) =$ 39.8 Greater than

MRAM $2(9)+2(6.5)+2(-7.6) = 15.8$ Don't know

TRAP $\frac{1}{2}(2)(2(8.3)+2(2.3)+9.3-10.5) = 20$ Don't know

No b/c f(x) lies below x-axis

 RRAM $\frac{\pi}{3}\left(\sin\frac{\pi}{3}+\sin\frac{2\pi}{3}+\sin\pi\right)=\frac{\pi\sqrt{3}}{3}$

7. Approximate the area of the region bounded by the graph of $y=\sin x$ and the x-axis from $x=0$ to $x=\pi$ using 3 equal subintervals using a) left endpoints, b) right endpoints, c) midpoints, and d) trapezoidal rule.

LRAM $\frac{\pi}{3}\left(\sin 0+\sin\frac{\pi}{3}+\sin\frac{2\pi}{3}\right)=\frac{\pi}{3}\left(0+\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)=\frac{\pi\sqrt{3}}{3}$

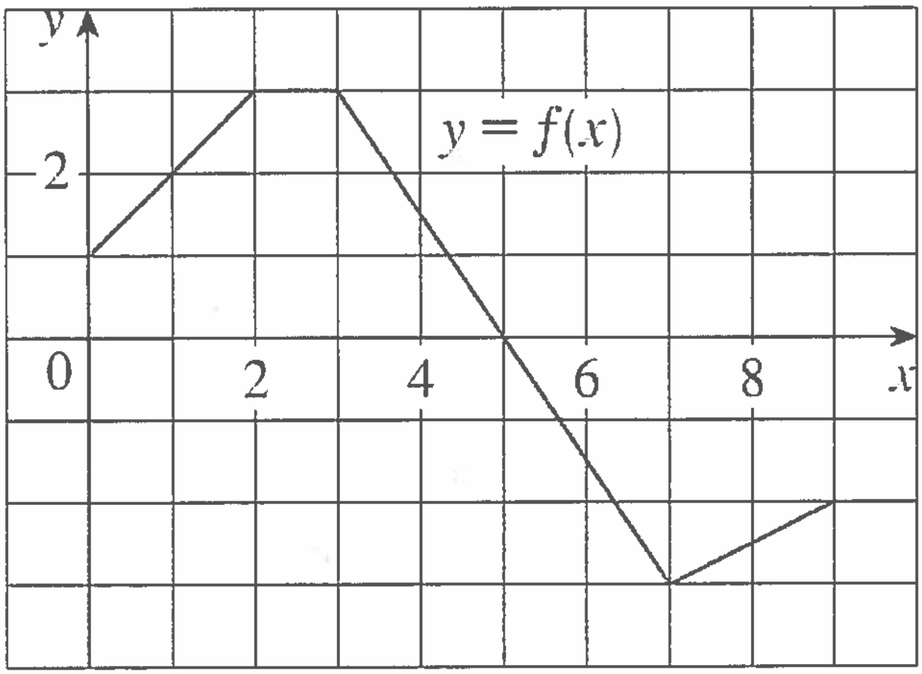
MRAM $=\frac{\pi}{3}\left(\sin\frac{\pi}{6}+\sin\frac{\pi}{2}+\sin\frac{5\pi}{6}\right)$

$\frac{\pi}{3}\left(\frac{1}{2}+1+\frac{1}{2}\right)=\frac{2\pi}{3}$

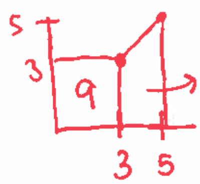
TRAP: $\frac{\pi\sqrt{3}}{3}$

8. The graph of f is shown below. Evaluate each integral by interpreting it in terms of areas.

a) $\int_0^2 f(x)dx$ 4 b) $\int_0^5 f(x)dx$ 10 c) $\int_5^7 f(x)dx$ -3 d) $\int_0^9 f(x)dx$ 2



9. Find $\int_0^5 f(x)dx$ if $f(x)=\begin{cases}3, & x<3\\ x, & x\geq 3\end{cases}$



$\frac{1}{2}(3+5)(2)$

$\frac{1}{2}(16)=8$

$9+8=17$

10. Given that $\int_4^9 \sqrt{x}dx=\frac{38}{3}$, what is

a) $\int_9^4 \sqrt{t}dt$ $-\frac{38}{3}$ b) $\int_4^9\left(\sqrt{x}+3\right)dx$ $\frac{83}{3}$ $3\times5=15$ $+\frac{38}{3}$ c) $\int_9^{14}\sqrt{x-5}dx$ $=\frac{38}{3}$ d) $\int_4^4 \sqrt{x}dx$ 0

11. If $f(x)$ is represented by the table below, approximate $\int_1^{9.6} f(x)dx$ using left-endpoint, right-endpoint, midpoint, and trapezoidal approximations. Use as many subintervals as the data allows.

x	1	2.5	4	6	8	8.8	9.6	10.4
$f(x)$	4	3	1	3	5	6	4	7

LRAM: $1.5(4)+1.5(3)+2(1)+2(3)+.8(5)+.8(6)=27.3$

RRAM $1.5(3)+1.5(1)+2(3)+2(5)+.8(6)+.8(4)=30$

 MRAM

Trap 28.65

$3(3)+4(3)+1.6(6)=30.6$

12. Write as a single integral in the form $\int_a^b f(x)dx$: $\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx$

$\hookrightarrow \int_{-2}^{5} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx$

$= \int_{-1}^{5} f(x)dx$

13. If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.6$, find $\int_1^4 2f(x)dx$ 16.8

14. If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$, find $\int_0^9 [2f(x)+3g(x)]$ 122

15. (Calculator Permitted) Use your calculator's fnInt(function to evaluate the following integrals. Report 3 decimals.

a) $\int_0^5 \frac{x}{x^2+4}dx \approx .991$

b) $3+2\int_0^{\pi/3} \tan x dx$

≈ 4.386

13.) $\int_1^5 f(x)dx = 12$ $\quad$ ~~$\int_1^4 f(x)dx = 12 - 3.6 = 8.4$~~

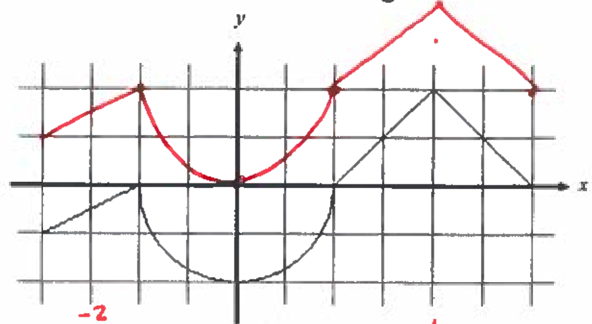
$\int_4^5 f(x)dx = 3.6$ $\quad$ $8.4 \cdot 2 = 16.8$

14.) $2(37)+3(16) = 74 + 48 = 122$

AP Calculus
5.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The graph of f below consists of line segments and a semicircle. Evaluate each definite integral.



a) $\int_0^2 f(x)\,dx = -\frac{1}{4}(\pi)(2)^2 = -\pi$

b) $\int_2^6 f(x)\,dx$ 4

c) $\int_{-4}^{2} f(x)\,dx = -\frac{1}{2}(2)(1) = -1-2\pi$

d) $\int_6^0 f(x)\,dx$ $\pi - 2$

e) $\int_{-4}^{6} |f(x)|\,dx$ $\frac{1}{2}(4)(2)=4$

$1+2\pi+4 = 5+2\pi$

f) $\int_{-4}^{6} [f(x)+2]\,dx$

$\int_{-4}^{-2} f(x) = 2+1 = 3$

$\int_2^6 F(x) = 12$

$\int_{-2}^{2} f(x) = 8-2\pi$

$\int_{-4}^{6} f(x) = 23-2\pi$

2. Part *e* above, gives a way to find the total <u>area</u> between the x–axis and the function between $x = -4$ and $x = 6$. *Without using absolute value signs*, write an expression that can be used to find the total <u>area</u> between the x–axis and the function between $x = -4$ and $x = 6$.

$-\int_{-4}^{2} f(x)\,dx + \int_2^6 f(x)\,dx$

3. Suppose that f and g are continuous and $\int_1^2 f(x)dx = -4$, $\int_1^5 f(x)dx = 6$, and $\int_1^5 g(x)dx = 8$.

Find each of the following:

a) $\int_2^2 g(x)dx$ 0

b) $\int_5^1 7g(x)dx$ $7 \cdot -8 = -56$

c) $\int_1^2 3f(x)dx$ $-4 \cdot 3 = -12$

$\int_2^5 f(x)dx + \int_1^2 f(x)dx = \int_1^5 f(x)dx$

d) $\int_2^5 f(x)dx = 10$

$6 - -4$

e) $\int_1^5 [f(x) - g(x)]dx$

$6-8 = -2$

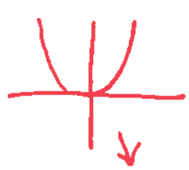
f) $\int_1^5 [9f(x)+4]dx$

$9(6) = 54$

$4 \times 4 = 16$

$54+16 = 70$

4. What are all the values of k for which $\int_2^k x^2\,dx = 0$?



A –2
B 0
C 2
D –2 and 2
E –2, 0, and 2

always + so no canceling

5. If $\int_3^7 f(x)dx = 5$ and $\int_3^7 g(x)dx = 3$, then all of the following must be true *except*

A $\int_3^7 f(x)g(x)dx = 15$ NO rule for multiplication of integrals

B $\int_3^7 [f(x)+g(x)]dx = 8$ ✓

C $\int_3^7 2f(x)dx = 10$ ✓

D $\int_3^7 [f(x)-g(x)]dx = 2$ ✓

E $\int_7^3 [g(x)-f(x)]dx = 2$ ✓ 3-5= -2 but backward limits so --2 =2

6. A driver average 30 mph on a 150-mile trip and then returned over the same 150 miles at the rate of 50 mph. He figured his average speed was 40 mph for the entire trip.

a) What was the total distance traveled?

300 miles

b) What was his total time spent for the trip?

5h + 3h = 8h

c) What was his average speed for the trip?

$\frac{300}{8} = 37.5$ mph

d) Explain the driver's error in reasoning.

you spend more time when you drive slower (30 vs 50) so it's a weighted avg.

7. A dam released 1000 m^3 of water at 10 m^3/min and then released another 1000 m^3 at 20 m^3/min. What was the average rate at which the water was released? Give reasons for your answer.

$\frac{1000m^3}{10\ m^3/min} = 100$ min $\quad \frac{1000m^3}{20m^3/min} = 50$ min

$\frac{2000m^3}{150min} = 13\frac{1}{3}\ m^3/min$

Avg rate = $\frac{\text{Total relased}}{\text{total time elapsed}}$

8. [Calculator] At different altitudes in Earth's atmosphere, sound travels at different speeds. The speed of sound $s(x)$ (in meters per second) can be modeled by

$$s(x) = \begin{cases} -4x+341 & \text{if } 0 \le x < 11.5 \\ 295 & \text{if } 11.5 \le x < 22 \end{cases}$$

where x is measured in kilometers. What is the average speed of sound over the interval [0. 22]?

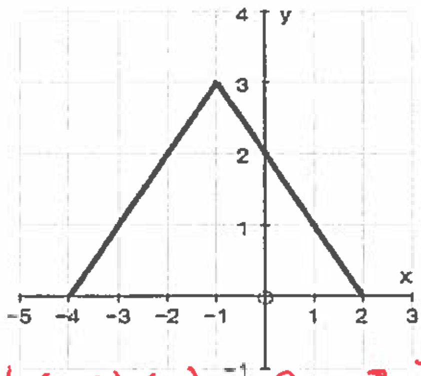
$$\frac{1}{22-0}\int_0^{22} s(x)dx = \frac{1}{22}\left[\int_0^{11.5} -4x+341\,dx + \int_{11.5}^{22} 295\,dx\right]$$

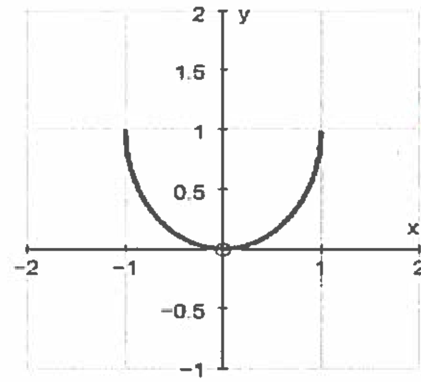
$$= \frac{3657 + 3097.5}{22} = 307.023\ m/s$$

9. Find the average value of the function on the interval *without integrating*, by appealing to the geometry of the region between the graph and the x-axis.

a) $f(x)=\begin{cases} x+4 & -4\le x\le -1 \\ -x+2 & -1<x\le 2\end{cases}$ on $[-4, 2]$

b) $f(x)=1-\sqrt{1-x^2}$ on $[-1, 1]$ $\frac{1}{2}\pi\cdot 1 = \frac{\pi}{2}$





$\frac{2-\frac{\pi}{2}}{2} = 1-\frac{\pi}{4}$

$\frac{1}{2}(6)(3)=9 \to \frac{9}{6}=1.5$

10. Set up an integral to find the average value of the functions in the last question, then use your calculator to evaluate.

a) $\frac{1}{6}\left[\int_{-4}^{-1}(x+4)\,dx+\int_{-1}^{2}(-x+2)\,dx\right]$

$=\frac{3}{2}$

b) $\frac{1}{2}\int_{-1}^{1}\left(1-\sqrt{1-x^2}\right)dx$

$\approx .2146 \to 1-\frac{\pi}{4}$

11. [Calculator] Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t)=82+4\sin\left(\frac{t}{2}\right) \quad \text{for } 0\le t\le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

a) Is traffic flow increasing or decreasing at $t=7$? Give a reason for your answer.

$F'(t)=\frac{1}{2}(4)\cos\left(\frac{t}{2}\right) \to 2\cos\left(\frac{t}{2}\right) \to 2\cos\left(\frac{7}{2}\right)=-1.87$

decreasing b/c r.o.c. is $(-)$.

b) What is the average value of the traffic flow over the time interval $10\le t\le 15$? Indicate units of measure.

$\frac{1}{5}\int_{10}^{15} 82+4\sin\left(\frac{t}{2}\right)dt \approx 81.899$ cars/min

c) What is the average rate of change of the traffic flow over the time interval $10\le t\le 15$? Indicate units of measure

$F(15)\approx 85.752$

$F(10)\approx 78.164$

$\frac{85.752-78.164}{15-10} \approx 1.518$ cars/min