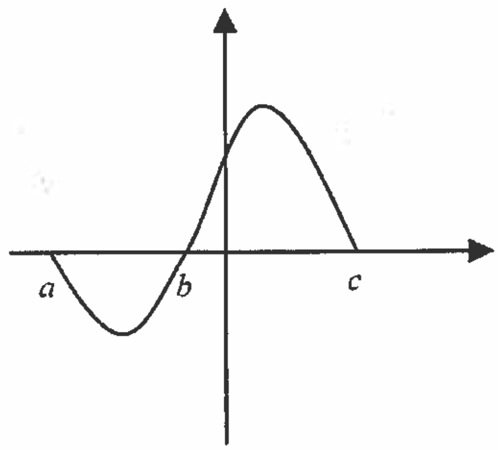
ABCALC Definite Integrals Homework Name: Key

1. Graphically speaking, if $f(x)$ is always above the x-axis, what does $\int_a^b f(x)dx$ mean?

The area between $f(x)$ and the x-axis from a to b.

2. Given the graph of $f(x)$ below, answer the following questions.



a) Is $\int_a^b f(x)dx$ positive, negative, or zero? Why?

neg. below x-axis

b) Is $\int_b^c f(x)dx$ positive, negative, or zero? Why?

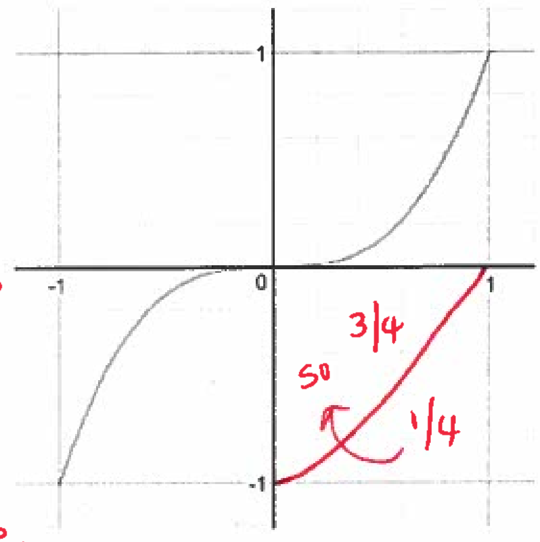
pos. above x-axis

c) Is $\int_a^c f(x)dx$ positive, negative, or zero? Why?

pos. more area above than below x-axis

$\boxed{x^3 \text{ is odd!}}$

3. The graph of $y = x^3$ is given below. Use it and the fact that $\int_0^1 x^3 dx = \frac{1}{4}$ to evalute each of the following.



part c

a) $\displaystyle\int_{-1}^{1} x^3 dx = 0$

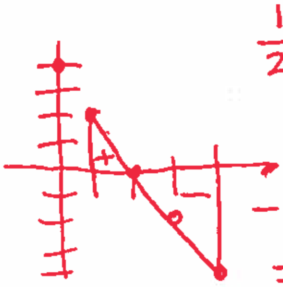
b) $\displaystyle\int_{0}^{1} (x^3 + 3)dx$ Same as $\int_0^1 x^3$ w/ a rectangle $\underset{1}{\boxed{}}\,3$, so 3.25

c) $\displaystyle\int_{0}^{1} (x^3 - 1)dx$

$-\frac{3}{4} \rightarrow$ picture

4. Draw a sketch and shade the area indicated by each integral, then use geometry to evalute the integral.

a) $\displaystyle\int_{1}^{4} (-2x + 4)dx$



$\frac{1}{2}(1)(2) = 1$

$-\frac{1}{2}(2)(4) = -4 \rightarrow -3$

b) $\displaystyle\int_{-4}^{0} \sqrt{16 - x^2}\, dx$

$y = \sqrt{16 - x^2}$

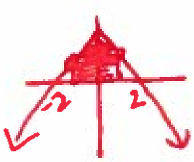
$y^2 = 16 - x^2$

$x^2 + y^2 = 16$

↳ but just +

$\frac{1}{4}\pi(4)^2 = 4\pi$

c) $\displaystyle\int_{-1}^{1} (2 - |x|)dx$

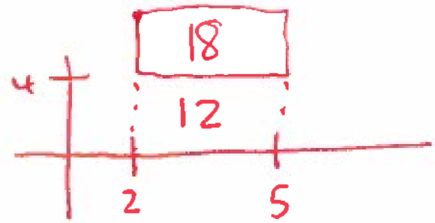


Triangle $= \frac{1}{2}(2)(1) = 1$

Rect. $= 2(1) = 2$

$1 + 2 = 3$

5. If $\int_2^{5} f(x)dx = 18$, then find $\int_2^5 (f(x) + 4)dx$

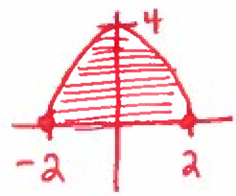


$= 30$

6. Draw a sketch for the area enclosed between the x-axis and the graph of $y = 4 - x^2$ over $[-2, 2]$. Set up an integral to find the area of the region and use your calculator to evaluate the integral.

$4 - (-2)^2 = 0$

$4 - 2^2 = 0$



$$\int_{-2}^{2} (4 - x^2)dx$$

$$= 10\tfrac{2}{3} = \frac{32}{3}$$

7. Approximate the area under the curve defined by $y = x^2 - 2x + 3$ from $[-1, 3]$ using the left rectangular approximation method with 4 subintervals of equal length.

$\frac{3--1}{4} = \frac{4}{4} = 1$

$1(6) + 1(3) + 1(2) + 1(3) = 14$

$f(-1) = 6 \quad f(0) = 3 \quad f(1) = 2 \quad f(2) = 3$

8. The function f is continuous over the closed interval $[0, 10]$ and has values that are given in the table.

x	0	2	5	7	10
$f(x)$	2	3	5	7	8

Using 4 subintervals, find each of the following approximations for the area under the curve from $[0, 10]$.

a) LRAM $2(2) + 3(3) + 2(5) + 3(7) = 44$

b) RRAM $2(3) + 3(5) + 2(7) + 3(8) = 59$

c) Trapezoid $\frac{1}{2}(2+3)(2) + \frac{1}{2}(3+5)(3) + \frac{1}{2}(5+7)(2)$

$+ \frac{1}{2}(7+8)(3) = 5 + 12 + 12 + 22.5$

$= 51.5$