AB Calculus The Derivative of a Function Part 3 Homework Name: Key

1. What words/phrases/formulas are associated with average rate of change?

Slope of secant line ; $\frac{f(b)-f(a)}{b-a}$; slope between 2 pts.

2. What words/phrases/formulas are associated with instantaneous rate of change?

Slope of tangent line ; $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$; $f'(x)$; derivative ; slope of curve @ 1 pt.

3. Use the original limit definition of derivative to find the derivative of each of the following.

a) $f(x) = 4x^3 + 2$

$$\lim_{h\to 0}\frac{4(x+h)^3+2-4x^3-2}{h}$$

$$\lim_{h\to 0}\frac{4x^3+12x^2h+12xh^2+4h^3-4x^3-2}{h}$$

$$\lim_{h\to 0} 12x^2+12xh+4h^2$$

$$12x^2$$

b) $f(x) = -\frac{1}{x-4}$

$$\lim_{h\to 0}\frac{-\frac{1(x-4)}{(x+h-4)(x-4)}+\frac{1}{x-4}\cdot\frac{(x+h-4)}{(x+h-4)}}{h} \to \lim_{h\to 0}\frac{-x+4+x+h-4}{h(x+h-4)(x-4)}$$

$$\lim_{h\to 0}\frac{h}{h(x+h-4)(x-4)} \to \frac{1}{(x-4)^2}$$

4. Use the alternative definition of the derivative to find the derivative of each of the following.

a) $f(x) = \sqrt{5x-4}$ at $x = 4$

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c} \to \lim_{x\to 4}\frac{\sqrt{5x-4}-\sqrt{5(4)-4}}{x-4} \to \frac{\sqrt{5x-4}-4}{x-4}\cdot\frac{\sqrt{5x-4}+4}{\sqrt{5x-4}+4}$$

$$\frac{5x-4-16}{(x-4)(\sqrt{5x-4}+4)} = \frac{5x-20}{(x-4)(\sqrt{5x-4}+4)} \to \frac{5(x-4)}{(x-4)(\sqrt{5x-4}+4)} = \frac{5}{\sqrt{5x-4}+4} \to \frac{5}{\sqrt{5\cdot4-4}+4} = \frac{5}{4+4} = \frac{5}{8}$$

b) $f(x) = 4x^2 + 2x + 1$ at $x = -1$.

$$\lim_{x\to -1}\frac{4x^2+2x+1-4+2-1}{x+1} \to \frac{4x^2+2x-2}{x+1} \to \frac{(4x-2)(x+1)}{x+1} = 4x-2$$

$4(-1)-2$

$-4-2$

-6

5. Write the equation of the tangent line and normal line to the function $f(x) = 4x - 5$ at $x = 3$.

$f'(x) = 4$ since $4x-5$ is linear $\qquad f(3) = 12-5 = 7$

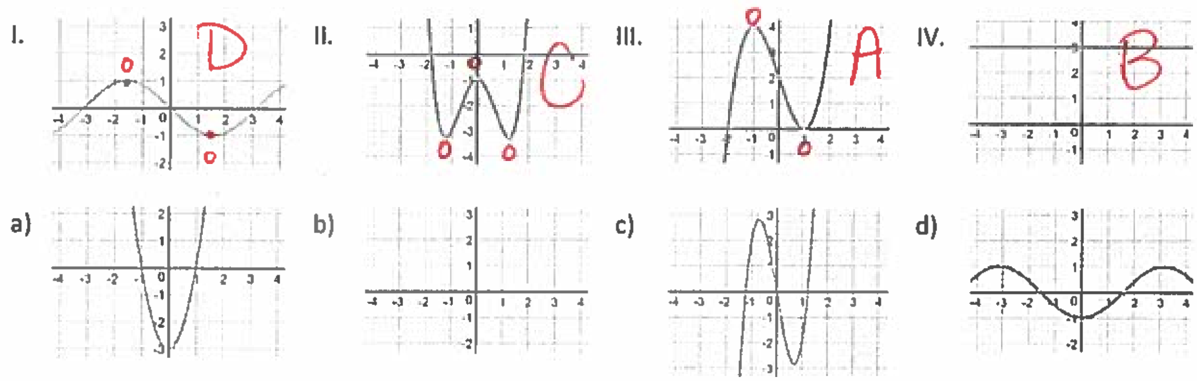
$f'(3) = 4$

Tangent: $y - 7 = 4(x-3)$ $\qquad$ Normal: $y - 7 = -\frac{1}{4}(x-3)$

6. If $f'(1) = -5$ and $f(1) = -3$, find the equation of the tangent line to $f(x)$ at $x = 1$.

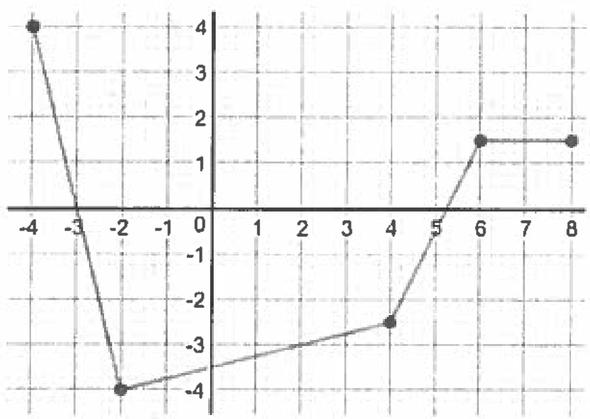
$y + 3 = -5(x-1)$

7. Match the graph of each function in the top row with the graph of its derivative in the bottom row.



8. The graph of the function $y = f(x)$ shown here is made of line segments joined end to end. Graph $f'(x)$ in the space provided below.

Graph of $f(x)$



Graph $f'(x)$ here