AB Calculus The Derivative of a Function Part 1 Homework Name: Key

1. What is the original limit definition of the derivative?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

2. What is the alternate definition of the derivative?

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x-c}$$

3. Use the original limit definition of derivative to find the derivative of each of the following.

a) $f(x) = \frac{1}{x}$ at $x = 2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \rightarrow \frac{\frac{x}{x}\cdot\frac{1}{(x+h)} - \frac{1}{x}\cdot\frac{(x+h)}{(x+h)}}{h} \rightarrow \frac{x - x - h}{x(x+h)} \cdot \frac{1}{h}$$

$$\rightarrow \frac{-h}{x(x+h)h} \rightarrow \frac{-1}{x(x+h)} \rightarrow \frac{-1}{x(x+0)} \rightarrow \boxed{\frac{-1}{x^2}} \ @\ x = 2 \rightarrow \boxed{-\frac{1}{4}}$$

b) $f(x) = x - x^3$ at $x = -1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \frac{(x+h) - (x+h)^3 - (x - x^3)}{h} \rightarrow$$

$$\frac{x+h - (x^3 + 3x^2h + 3xh^2 + h^3) - x + x^3}{h} \rightarrow \frac{x+h - x^3 - 3x^2h - 3xh^2 - h^3 - x + x^3}{h}$$

$$\rightarrow \frac{h - 3x^2h - 3xh^2 - h^3}{h} \rightarrow 1 - 3x^2 - 3xh - h^2 \rightarrow \boxed{1 - 3x^2} \ @\ x = -1 \quad 1 - 3(-1)^2 \boxed{= -2}$$

4. Use the alternative definition of the derivative to find the derivative of each of the following.

a) $f(x) = \sqrt{x+1}$ at $x = 3$

$$f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x-3} \Rightarrow \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3} \rightarrow \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} \rightarrow \frac{x+1-4}{x-3(\sqrt{x+1}+2)}$$

$$\rightarrow \frac{x-3}{x-3(\sqrt{x+1}+2)} \rightarrow \frac{1}{\sqrt{x+1}+2} \rightarrow \frac{1}{\sqrt{3+1}+2} \rightarrow \frac{1}{2+2} \rightarrow \boxed{\frac{1}{4}}$$

b) $f(x) = 3 - x^2$ at $x = -2$

$$f'(-2) = \lim_{x\to -2} \frac{f(x) - f(-2)}{x+2} \rightarrow \frac{3 - x^2 - (3 - (-2)^2)}{x+2} \rightarrow \frac{3 - x^2 - 3 + 4}{x+2}$$

$$\rightarrow \frac{-x^2 + 4}{x+2} \rightarrow \frac{4 - x^2}{x+2} \rightarrow \frac{(2-x)(2+x)}{x+2} \rightarrow 2 - x \rightarrow 2 - (-2) \boxed{= 4}$$

5. Consider the function $g(x) = 10$. → Horizontal Line

a) Using what you know about the graph of $g(x) = 10$, what is $g'(6)$? $g'(6) = 0$

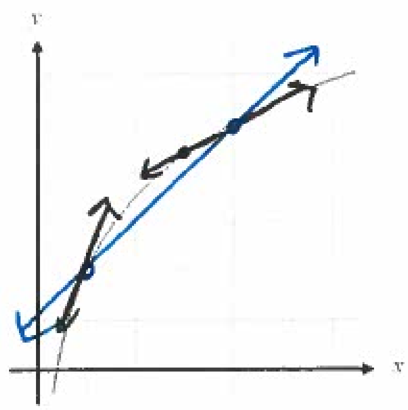
b) Use the original limit definition of the derivative to verify your answer for part a.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \to \frac{10 - 10}{h} = \frac{0}{h} = 0$$

6. If $f(2) = 3$ and $f'(2) = 5$, find the equation of the tangent line to $f(x)$ at $x = 2$.

$y - y_1 = m(x - x_1)$ $\quad$ $y - 3 = 5(x - 2)$

7. Use the figure to the right to answer the following questions.



a) Find $f(1)$ and $f(4)$.

$f(1) = 2$ $\quad$ $f(4) = 5$

b) What is the geometric interpretation of $\frac{f(4)-f(1)}{4-1}$? Draw it on the graph to the right.

the slope of the secant line between $(1,2)$ & $(4,5)$ ↔ also called average rate of change

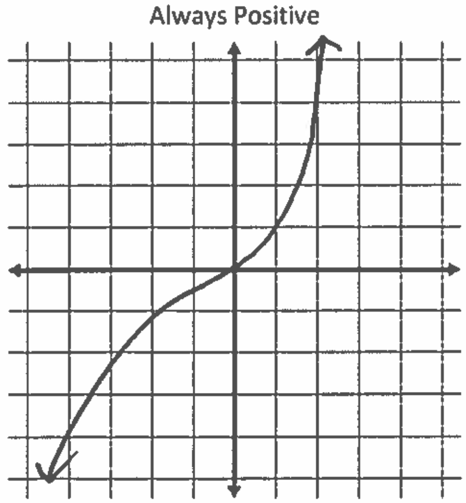
c) Using the geometric interpretation of each expression, insert the inequality symbol (< or >) in the box between the two expressions that makes the statement true.

$\left(\frac{3}{3}\right) \frac{f(4) - f(1)}{4 - 1} \boxed{>} \frac{f(4) - f(3)}{4 - 3} \left(\frac{1/2}{1}\right)$ $\qquad$ $\left(\frac{3}{3}\right) \frac{f(4) - f(1)}{4 - 1} \boxed{<} f'(1)$ ← larger positive b/c steeper

↳ steeper

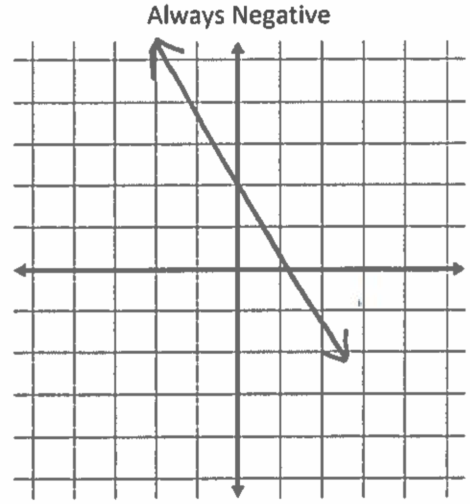
8. Sketch a function whose derivative is always positive and another whose derivative is always negative.

Always Positive



Always increasing

Always Negative



Always decreasing