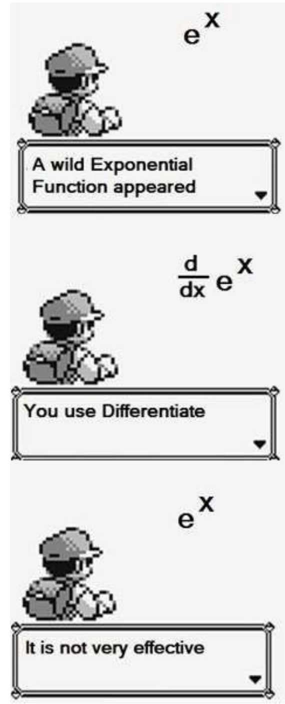
AB Calculus: Derivatives of Exp. and Log Functions **Name:** ____________________



Derivative of e^x

$$\frac{d}{dx}[e^x] = e^x$$

The Chain Rule and e^x

If u is a differentiable function of x, then

$$\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}$$

Example 1 $\frac{d}{dx}[e^{2x-1}]$

$= e^{2x-1} \quad 2$

Example 2 $\frac{d}{dx}\left[e^{\frac{3}{x}}\right]$ $\quad e^{3x^{-1}}$

$= e^{3x^{-1}} \cdot -3x^{-2}$

$y' \to \frac{dy}{dx}$

Example 3 Find y' if $y^2 + e^y = 2x^2$

$\frac{d}{dx}\left[y^2 + e^y = 2x^2\right]$

$2y\frac{dy}{dx} + e^y\frac{dy}{dx} = 4x$

$\frac{dy}{dx}(2y + e^y) = 4x$

$\boxed{\frac{dy}{dx} = \frac{4x}{2y + e^y}}$

Derivative of $\ln x$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$\frac{d}{dx}[\ln x]$

$\frac{1}{x}$

The Chain Rule and $\ln x$

If u is a differentiable function of x, then

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}$$

Example 4 prove $\frac{d}{dx}[\ln x] = \frac{1}{x}$ using implicit differentiation

$y = \ln x$

$e^\wedge \quad e^\wedge$

$\frac{d}{dx}\left[e^y = x\right]$

$e^y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$

$y' = \frac{dy}{dx}$ **Example 4** Find y' if $\frac{d}{dx}\left[y = \ln(2x + 2) \right]$

$$\frac{dy}{dx} = \frac{1}{2x+2}(2) = \frac{2}{2x+2} = \frac{1}{x+1}$$

Example 5 Let $f(x) = \ln(\tan x)$. Find $f'(x)$.

$$f'(x) = \frac{1}{\tan x} \cdot \sec^2 x$$

$$= \frac{\sec^2 x}{\tan x}$$

Logarithmic Differentiation

The properties of logarithms can be used to simplify some problems. Here is a review of the properties

Name	**Mathematical Property**	**Example**
Definition of Logarithm	If $b^c = a$, then $log_b a = c$	If $2^4 = 16$, then $log_2 16 = 4$
Addition Rule	$log_b(MN) = log_b(M) + log_b(N)$	$log_2(5x) = log_2(5) + log_2(x)$
Subtraction Rule	$log_b\left(\frac{M}{N}\right) = log_b(M) - log_b(N)$	$log_2\left(\frac{5}{x}\right) = log_2(5) - log_2(x)$
Exponent Rule	$log_b(M^k) = k \cdot log_b(M)$	$log_2(5^3) = 3 \cdot log_2(5)$
Change of Base	$log_b a = \frac{\ln a}{\ln b}$	$log_2 3 = \frac{\ln 3}{\ln 2}$

Example 6 Rewrite $f(x)$ using properties of logs and find $f'(x)$

$f(x) = log_5\sqrt{x} \rightarrow f(x) = \frac{\ln\sqrt{x}}{\ln 5} = \frac{1}{\ln 5}\ln\sqrt{x}$

$= \frac{1}{\ln 5}\ln x^{\frac{1}{2}} \rightarrow \frac{1}{2\ln 5}\ln x = f(x)$

$$f'(x) = \frac{1}{2\ln 5} \cdot \frac{1}{x}$$

Example 7 Use the properties of logarithms to rewrite $f(x)$ and find $f'(x)$ in terms of x. Also find $f'\left(\frac{\pi}{2}\right)$

$$f(x) = x^{\sin(x)}$$

$$\ln y = \ln x^{\sin x}$$

$$\frac{d}{dx}\left[\ln y = \sin x \cdot \ln x\right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \ln x + \sin x \cdot \frac{1}{x}$$

Just cross multiply by y

$$\frac{dy}{dx} = \left(\cos x \cdot \ln x + \frac{\sin x}{x}\right) \cdot y$$

$$f'\left(\tfrac{\pi}{2}\right) = \left(\cos\left(\tfrac{\pi}{2}\right)\ln\left(\tfrac{\pi}{2}\right) + \frac{\sin(\pi/2)}{\pi/2}\right)\left(\tfrac{\pi}{2}\right)^{\sin\left(\frac{\pi}{2}\right)}$$

$$= \left(0 + \tfrac{2}{\pi}\right)\left(\tfrac{\pi}{2}\right) = \boxed{1 = f'\left(\tfrac{\pi}{2}\right)}$$

By utilizing the rules of logarithms and implicit differentiation, you can turn an exponential equation into an equation involving logarithms that is usually easier to deal with.

Example 9 $\frac{d}{dx}[2^x]$

$y = 2^x$

$\ln y = x \ln 2$

$\frac{1}{y} \frac{dy}{dx} = \ln 2$

$\frac{dy}{dx} = y \ln 2$

$\boxed{\frac{dy}{dx} = 2^x \ln 2}$

Example 10 $\frac{d}{dx}[3^x]$

$\boxed{\frac{dy}{dx} = 3^x \ln 3}$

Derivative of a^x where a is a constant

$$\frac{d}{dx}[a^x] = \ln a \cdot a^x$$

The Chain Rule and a^x where a is a constant

If u is a differentiable function of x, then

$$\frac{d}{dx}[a^u] = \ln a \cdot a^u \cdot \frac{du}{dx}$$

Example 11 Find the derivative of $f(x) = e^{5x} + 7^{2x} + \ln(x^2 + 4)$

$f'(x) = e^{5x} \cdot 5 + 7^{2x}\, 2 \ln 7 + \frac{1}{x^2+4}\, 2x$

Example 12 Find the derivative of $f(x) = e^{\tan 3x} + 6^{x^2} + \ln(\sec x)$

$f'(x) = e^{\tan(3x)} \sec^2(3x)\, 3 + 6^{x^2}\, 2x \cdot \ln 6 + \frac{1}{\sec x} \sec x \cdot \tan x$