Major Assessment Review Session Problems

Key

1. Find the derivative

a) $y = \dfrac{e^x}{4+x}$

$$\frac{dy}{dx} = \frac{(4+x)e^x - e^x(1)}{(4+x)^2}$$

b) $y = 2^{\frac{4}{x}} \rightarrow y = 2^{4x^{-1}}$

$$\frac{dy}{dx} = 2^{\frac{4}{x}} \cdot \ln 2 \cdot \frac{-4}{x^2}$$

c) $y = \ln(\sin x)$

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$$

d) $y = \ln\left(\dfrac{2x+6}{2x-9}\right)$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{2x+6}{2x-9}\right)} \cdot \left(\frac{(2x-9)(2) - (2x+6)(2)}{(2x-9)^2}\right)$$

$$= \frac{2x-9}{2x+6}\left(\frac{4x-18-4x-12}{(2x-9)^2}\right)$$

$$= \frac{-30}{(2x+6)(2x-9)}$$

2. If $f(x) = x^4 + 2x^3 - x^2 + 5$ and $f(2) = 33$, find $(f^{-1})'(33)$.

input of f

output of f

input of f^{-1}

$$\underbrace{(f^{-1})'(\;)}_{\text{input of } f^{-1}} = \frac{1}{f'(\underbrace{\;}_{\text{input of } f})}$$

$f'(x) = 4x^3 + 6x^2 - 2x$

$f'(2) = 4(2)^3 + 6(2)^2 - 2(2)$

$= 32 + 24 - 4$

$= 56 - 4 = 52$

$$(f^{-1})'(33) = \frac{1}{52}$$

Find the derivative of each of the following.

$3y^2y' + 2yy' - 5y' - 2x = 0$

1. $y^3 + y^2 - 5y - x^2 = -4$

$y' = \dfrac{2x}{3y^2 + 2y - 5}$

$\cos x - 2\sin 2y \cdot 2y' = 0$

2. $sinx + 2\cos(2y) = 1$

$y' = \dfrac{-\cos x}{-4\sin 2y}$

$x + x\tan y$

3. $sinx = x(1 + tany)$

$\cos x = 1 + \tan y + x\sec^2 y \cdot y'$

$y' = \dfrac{\cos x - 1 - \tan y}{x\sec^2 y}$

4. $x^3y^3 = x$

$3x^2y^3 + x^3 \cdot 3y^2y' = 1$

$y' = \dfrac{1 - 3x^2y^3}{3x^3y^2}$

5. $e^y = \ln(x) + sin^3(\sqrt{3x - 5})$

$x^3 \to 3x^2$
$\sin x \to \cos x$
$\sqrt{x} \to \frac{1}{2\sqrt{x}}$
$3x - 5 \to 3$

$e^y \cdot y' = \dfrac{1}{x} + 3(\sin\sqrt{3x-5})^2\cos\sqrt{3x-5} \cdot \dfrac{1}{2\sqrt{3x-5}} \cdot 3$

$y' = \left(\dfrac{1}{x} + \dfrac{9\sin^2\sqrt{3x-5}\cos\sqrt{3x-5}}{2\sqrt{3x-5}}\right) \cdot \dfrac{1}{e^y}$

6. Find the derivative

a) $x^3 - 3x^2y + 4y = 12$

$3x^2 - 6xy - 3x^2y' + 4y' = 0$

$y' = \dfrac{-3x^2 + 6xy}{-3x^2 + 4}$

b) $\sin x + 2\cos 2y = 1$

$\cos x - 2\sin 2y \cdot 2y' = 0$

$y' = \dfrac{-\cos x}{-4\sin 2y}$

c) $y = \sqrt{\cos^{-1} x}$

$\sqrt{x} \to \frac{1}{2\sqrt{x}}$

$\cos^{-1} x \to \frac{-1}{\sqrt{1-x^2}}$

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{\cos^{-1} x}} \cdot \dfrac{-1}{\sqrt{1-x^2}}$

d) $y = \sin(\tan^{-1} x)$

$\sin x \to \cos x$

$\tan^{-1} x \to \frac{1}{1+x^2}$

$\dfrac{dy}{dx} = \cos(\tan^{-1} x) \cdot \dfrac{1}{1+x^2}$

ABCALC Applications of Derivatives Review **Name:** ______________________________

1. Find the absolute extrema of the function and where they occur.

a) $f(x) = 4x^2 - 4x - 3$ over $[-2, 2]$

max? min max?

$[\;-\;|\;+\;]$

$-2 \quad \frac{1}{2} \quad 2$

$f'(x) = 8x - 4$

$0 = 8x - 4$

$x = \frac{1}{2}$

abs. min @ $x = \frac{1}{2} \to -4$

abs max @ $x = -2 \to 21$

-2	21 ✓
2	5

b) (Calculator) $f(x) = (x^2 - 9x)^{\frac{1}{3}}$ over $[-4, 8]$

$f'(x) = \frac{1}{3}(x^2 - 9x)^{-2/3}(2x - 9)$

CN: $0, 9, \frac{9}{2}$

max? min max?

$[\;-\;|\;-\;|\;+\;]\;|$

$-4 \quad 0 \quad 4.5 \quad 8 \quad 9$

	in calc
-4	3.733
0	0
4.5	-2.726
8	-2

abs max = 3.733 @ $x = -4$

abs min = -2.726 @ $x = 4.5$

2. Determine if MVT applies to the function. If it does, find the value of c guaranteed by the theorem. If it does not, explain why.

a) $f(x) = 4x^2 + 5x$ over $[-2, 1]$ Cont: $[-2, 1]$ differentiable: $(-2, 1)$ ✓

$f(1) = 9 \quad f(-2) = 6$

$f'(x) = 8x + 5$

$\frac{6 - 9}{-2 - 1} = \frac{-3}{-3} = 1$

$8x + 5 = 1 \quad 8x = -4 \quad x = -\frac{1}{2} \quad c = -\frac{1}{2}$

b) (Calculator) $f(x) = \sin x$ over $[4, 5]$ cont: $[4, 5]$ diff: $(4, 5)$ ✓

$\frac{\sin 5 - \sin 4}{5 - 4} \approx -.202$

$f'(x) = \cos x$

$\cos x = -.202 \to$ put in graph find intersection or

By unit circle

$x \approx 1.774$ (Q2)

also Q4 → Ref ∠ = $\pi - 1.774$

$= 1.367$

Q3: $\pi + 1.367$

$c = 4.509$

3. For $f(x) = x^4 - 12x^2 - 13$, find the following.

a) Find the intervals that $f(x)$ is increasing and decreasing. Justify your response.

$f'(x) = 4x^3 - 24x$

$0 = 4x(x^2 - 6)$ CN: $0, \pm\sqrt{6}$

$-\;|\;+\;|\;-\;|\;+$

$-\sqrt{6} \quad 0 \quad \sqrt{6}$

Inc: $(-\sqrt{6}, 0)\,(\sqrt{6}, \infty)$ $f'(x) > 0$

Dec: $(-\infty, -\sqrt{6})\;(0, \sqrt{6})$ $f'(x) < 0$

b) Find the x-values of all local minimum and maximum values. Justify each answer.

Local min @ $x = \pm\sqrt{6}$ $f'(x)$ changes − to +

Local max @ $x = 0$ $f'(x)$ changes + to −

c) Find all points of inflection. Justify your response.

$f''(x) = 12x^2 - 24$

$0 = 12(x^2 - 2)$

$x = \pm\sqrt{2}$

$+\;|\;-\;|\;+$

$-\sqrt{2} \quad \sqrt{2}$

P.O.I @ $x = \pm\sqrt{2}$

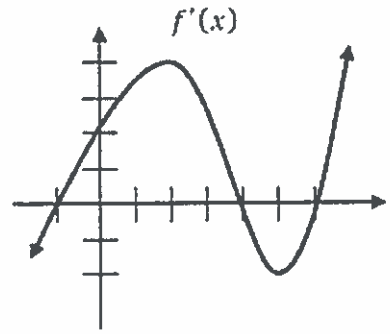
$f''(x)$ changes signs

d) Find the intervals that $f(x)$ is concave up and concave down. Justify your response.

CCU: $(-\infty,-\sqrt{2})$ $(\sqrt{2},\infty)$ $f''(x)>0$

CCD: $(-\sqrt{2},\sqrt{2})$ $f''(x)<0$

4. Use the graph of $f'(x)$ to the right to answer the following. Justify each response.



Note: Graph of f'(x) not $f(x)$.

a) What is the slope of $f(x)$ at $x = 2$?

4 $f'(2)=4$

b) For which x values does $f(x)$ have a horizontal tangent line?

$x=-1, 4, 6$ $f'(x)=0$

c) Find the intervals where $f(x)$ is increasing.

$(-1,4)$ $(6,\infty)$ $f'(x)>0$

d) Find the x-values where $f(x)$ has a relative minimum/maximum.

rel min @ $x=-1$, $x=6$ $f'(x)$ changes $-$ to $+$

rel max @ $x=4$, $f'(x)$ changes $+$ to $-$

e) Is $f(x)$ increasing or decreasing at $x = 5$?

$f'(5)=-2$ so $f(x)$ is decreasing @ $x=5$

f) Is $f(x)$ concave up or concave down at $x = 4$?

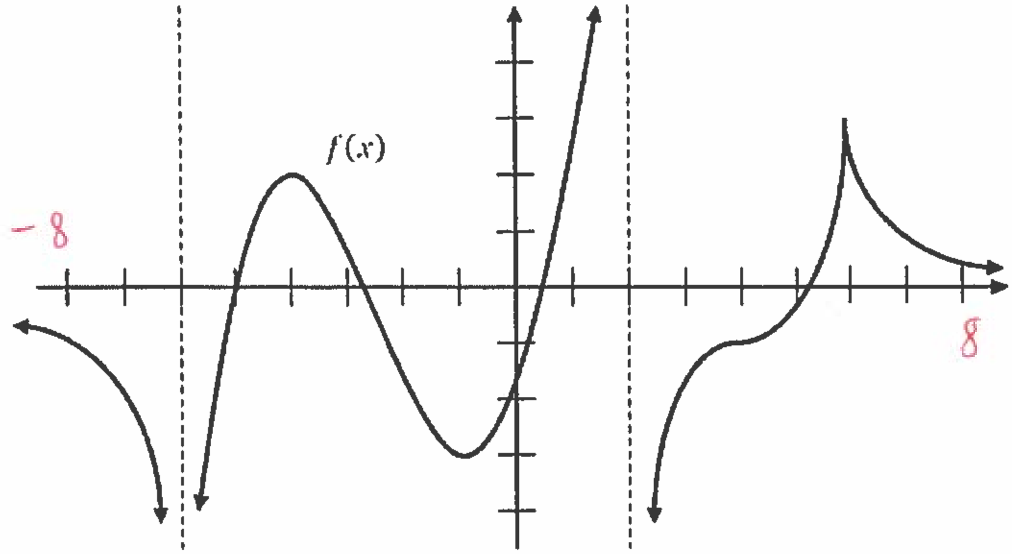
$f'(x)$ is decreasing @ $x=4$, so $f(x)$ is CCD

f) Find the x-values where $f(x)$ has a point of inflection.

$x=2$, $x=5$ $f'(x)$ changes direction

($f''(x)$ changes signs)

5. Use the graph of $f(x)$ below to answer the following



a) Find the intervals where $f(x)$ is increasing and decreasing.

Inc: $(-6,-4)$ $(-1,2)$ $(2,4)$ $(4,6)$ / Dec: $(-\infty,-6)$ $(-4,-1)$ $(6,\infty)$

b) Find all x-values where the slope of $f(x)$ is zero.

$x=-4,-1,4$

c) Find all x-values where the derivative of $f(x)$ does not exist

$x=-6,2,6$

d) Find all critical points of $f(x)$

$x=-6,2,6 \rightarrow f'(x)$ und AND $x=-4,-1,4 \rightarrow f'(x)=0$

e) Find all coordinates where $f(x)$ has relative extrema.

$x=-4,-1,6$

f) Find all x-values where $f(x)$ has a point of inflection (approximate if necessary)

$x=-2.5, 4$

g) Find the intervals where $f(x)$ is concave up and concave down (approximate if necessary)

CCU: $(-2.5,2)$ $(4,6)$ $(6,\infty)$ / CCD. $(-\infty,-6)$ $(-6,-2.5)$ $(2,4)$

6. Find the points of horizontal AND vertical tangency for $f(x)=\sqrt{9-x^2}$ $(9-x^2)^{1/2}$

$$f'(x)=\frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)$$

$$=\frac{-2x}{2\sqrt{9-x^2}}$$

$f(0)=\sqrt{9}=3$ H.T: $(0,3)$

$f(\pm 3)=\sqrt{9-9}=0$ V.T: $(\pm 3,0)$

H.T.: $-2x=0$ $x=0$

V.T: $9-x^2=0$ $x=\pm 3$

7. If $f(x) = e^{2x}$, find the equation of the tangent line to $f(x)$ at the point $\left(\frac{1}{2}\ln 5, 5\right)$.

$f'(x) = 2e^{2x}$

$f'\left(\frac{1}{2}\ln 5\right) = 2e^{2\left(\frac{1}{2}\right)\ln 5}$

$= 2e^{\ln 5}$

$= 2 \cdot 5$

$= 10$

$y - 5 = 10\left(x - \frac{1}{2}\ln 5\right)$

8. Use the information in the table about $f(x)$ over $[-3, 6]$ to answer the following questions.

x	-3	$-3 < x < 0$	0	$0 < x < 3$	3	$3 < x < 6$	6
f	-4	$-$	0	$-$	-2	$-$	0
f'	10	$+$	0	$-$	DNE	$+$	2
f''	$-$	$-$	$-$	$-$	$-$	$-$	$-$

INC/CCD max DEC/CCD min INC/CCD

2nd Derivative Test

$f'(x) = 0$ & $f''(x) < 0$

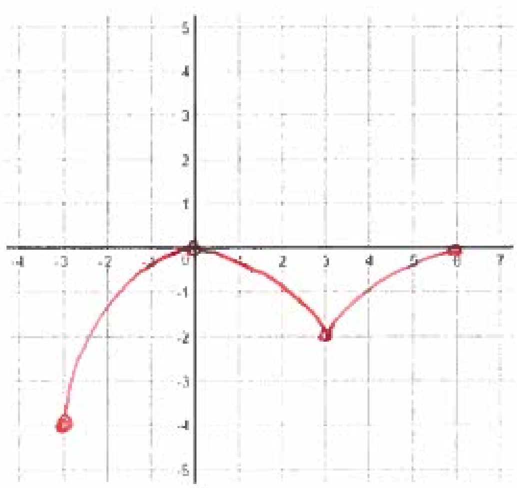
a) Find the points of relative and absolute extrema for $f(x)$. Justify your response.

→ rel max $= 0$ @ $x = 0$ & rel min $= -2$ @ $x = 3$

or ↳ $f'(x)$ changes + to − ↳ $f'(x)$ changes − to +

abs max $= 0$ @ $x = 0, 6$ abs min $= -4$ @ $x = -3$

b) Sketch a graph of $f(x)$ on the axes below.



explanation for absolutes ↓ (table is fine)

x	$f(x)$	
-3	-4	→ min
0	0	→ max
3	-2	→ neither
6	0	→ max