AB Calculus Derivatives of Logs and Exponential Functions Homework Name: Key

1. Find d^2y/dx^2 if $y^3 + y = 2\cos x$

$3y^2y' + y' = -2\sin x$

$y' = \frac{-2\sin x}{3y^2+1}$

$y'' = \frac{(3y^2+1)(-2\cos x) - (-2\sin x)(6yy')}{(3y^2+1)^2}$

LCD

$\frac{3y^2+1}{3y^2+1} \cdot \frac{-2(3y^2+1)\cos x + 12(\sin x)y\left(-\frac{2\sin x}{3y^2+1}\right)}{(3y^2+1)^2}$

$y'' = \frac{-2(3y^2+1)^2\cos x - 24y\sin^2 x}{(3y^2+1)^3}$ Simplified

2. Find dy/dx.

a) $y = 2e^x$

$y' = 2e^x$

b) $y = e^{-x}$

$y' = -e^{-x}$

c) $y = e^{\frac{2x}{3}}$

$e^x \to e^x$ $\frac{2x}{3} \to 2/3$

$e^{\frac{2}{3}x} \cdot \frac{2}{3}$

d) $y = xe^2 - e^x$

$e^2 - e^x$

e) $y = e^{\sqrt{x}}$

$e^x \to e^x$ $\sqrt{x} \to \frac{1}{2\sqrt{x}}$

$\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

f) $y = x^\pi$

$\pi x^{\pi-1}$

g) $y = x^{-\sqrt{2}}$

$-\sqrt{2}x^{-\sqrt{2}-1}$

h) $y = 8^x$

$\ln 8 \cdot 8^x$

i) $y = 3^{\csc x}$ take ln 1st

$\ln y = \csc x \ln 3$

$\frac{y'}{y} = -\ln 3 \csc x \cot x$

$y' = 3^{\csc x}(-\ln 3 \cdot \csc x \cot x)$

j) $y = x^{\ln x}$ take ln 1st

$\ln y = \ln x \ln x$ $(\ln x)^2$

$\frac{y'}{y} = \frac{2\ln x}{x}$

$y' = \frac{2x^{\ln x}\ln x}{x}$

k. $y = \ln(x^2)$

$\frac{1}{x^2} \cdot 2x = \frac{2}{x}$

l. $y = \ln\left(\frac{1}{x}\right)$

$\frac{1}{\frac{1}{x}}(-1x^{-2}) = \frac{-x}{x^2} = \frac{-1}{x}$

x^{-1}

m. $y = \ln(x+2)$

$\frac{1}{x+2}$

n. $y = \ln(2 - \cos x)$

$\frac{1}{2-\cos x} \cdot \sin x$

o. $y = \ln(\ln(x))$

$\ln x \quad \ln v$

$\frac{1}{x} \quad \frac{1}{x}$

$\frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$

p. $y = \log_4 x^2$

$4^y = x^2$

$y' \cdot \ln 4 \cdot 4^y = 2x$

$y' = \frac{2x}{\ln 4 \cdot x^2} = \frac{2}{x \ln 4}$

$= \frac{2}{x \ln 4}$

q. $y = \log_2(3x+1)$

$2^y = 3x+1$

$y' \ln 2 \cdot 2^y = 3$

$y' = \frac{3}{\ln 2 \cdot (3x+1)}$

r. $y = \log_2\left(\frac{1}{x}\right)$

$2^y = \frac{1}{x}$

$y' \cdot \ln 2 \cdot 2^y = \frac{-1}{x^2}$

$y' = \frac{-1}{x^2} \cdot \frac{x}{1} \cdot \frac{1}{\ln 2}$

$= -\frac{1}{x \ln 2}$

s. $y = \ln 2 \cdot \log_2 x$

Better cause faster

$\frac{\ln x}{\ln 2} \to$ faster!

$y = \ln 2 \cdot \frac{\ln x}{\ln 2}$

$y = \ln x \quad y' = \frac{1}{x}$

$y = \log_2 x^{\ln 2}$

$2^y = x^{\ln 2}$

$y' \cdot 2^y \cdot \ln 2 = \ln 2 \, x^{(\ln 2 - 1)}$

$y' = \frac{\ln 2 \cdot x^{(\ln 2)-1}}{\ln 2 \, x^{\ln 2}}$

$y' = x^{-1} = \frac{1}{x}$

t. $y = \log_{10} e^x$

$y = \frac{\ln e^x}{\ln 10}$

$y = \frac{x}{\ln 10} \Rightarrow y' = \frac{1}{\ln 10}$

3. Use the technique of logarithmic ~~differentiation to~~ find dy/dx in terms of x.

take ln 1st

a. $y = (\sin x)^x$

$\ln y = x \ln(\sin x)$

$\frac{y'}{y} = \frac{x \cdot \cos x}{\sin x} + \ln(\sin x)$

$y' = (\sin x)^x \left(\frac{x \cdot \cos x}{\sin x} + \ln \sin x \right)$

b. $y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$

$\left(\frac{(x-3)^4 \cdot (x^2+1)}{(2x+5)^3}\right)^{\frac{1}{5}} = \ln\left(\frac{(x-3)^{\frac{4}{5}} \cdot (x^2+1)^{\frac{1}{5}}}{(2x+5)^{\frac{3}{5}}}\right)$

$\ln y = \frac{4}{5}\ln(x-3) + \frac{1}{5}\ln(x^2+1) - \frac{3}{5}\ln(2x+5)$

$\frac{y'}{y} = \frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{3 \cdot 2 = 6}{5(2x+5)}$

$y' = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}} \left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)} \right)$