ABCALC Derivatives of Inverse Functions Homework

Name: Key

1. Find the derivative of each of the following

a) $y = \sin^{-1}(x^2)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^4}} \cdot 2x$

b) $y = (\sin^{-1} x)^2$

$\frac{dy}{dx} = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}$

c) $f(x) = \tan^{-1}(5x)$

$f'(x) = \frac{1}{1+(5x)^2} \cdot 5$

d) $f(x) = x^2 \arctan x$

$f'(x) = x^2 \cdot \frac{1}{1+x^2} + 2x \cdot \arctan x$

e) $f(x) = x\arcsin(1-x^2)$

$f'(x) = x \cdot \frac{1}{\sqrt{1-(1-x^2)^2}} \cdot -2x + 1 \cdot \arcsin(1-x^2)$

f) $y = \sec^{-1}(7x)$

$y' = \frac{1}{|7x|\sqrt{(7x)^2-1}} \cdot 7$

2. Find the derivative of the inverse function at the indicated point.

a) If $f(x) = x^3 + 7x + 9$, find $(f^{-1})'(1)$

$f'(x) = 3x^2 + 7$

$1 = x^3 + 7x + 9$

$0 = x^3 + 7x + 8$

$x = -1$

$f^{-1}(1) = \frac{1}{f'(-1)} = \frac{1}{3(-1)^2+7} = \frac{1}{10}$

or $x = y^3 + 7y + 9 \quad 1 = 3y^2y' + 7y' \rightarrow 1 = 3(-1)^2y' + 7y' \rightarrow 1 = 10y'$

$y' = \frac{1}{10}$

b) If $f(4) = 5$, and $f'(4) = \frac{2}{3}$, find $(f^{-1})'(5)$

$(f^{-1})'(5) = \frac{1}{f'(4)} = \frac{1}{2/3} = \frac{3}{2}$

3. Find the derivative of each of the following.

a) $x^2 + y^2 = 25$

$2x + 2yy' = 0$

$2yy' = -2x \quad y' = \frac{-2x}{2y} = \frac{-x}{y}$

b) $\frac{1}{x} + \frac{1}{y} = 1$

$x^{-1} + y^{-1} = 1$

$-\frac{1}{x^2} - \frac{1}{y^2} \cdot y' = 0$

$y' = \frac{1}{x^2} \cdot -y^2$

c) $(7x-1)^3 = 2y^4$

$3(7x-1)^2(7) = 8y^3y'$

$y' = \frac{21(7x-1)^2}{8y^3}$

d) $3x^2 - xy + 4y^2 = 90$

$6x - xy' - 1 \cdot y + 8yy' = 0$

$-xy' + 8yy' = -6x + y$

$y' = \frac{-6x+y}{-x+8y}$

e) $x^2y^3 - 5xy^2 - 4y = 4$

$2xy^3 + x^2\cdot 3y^2\cdot y' - (5x)(2y)y' - 5y^2 - 4y' = 0$

$3x^2y^2y' - 10xyy' - 4y' = -2xy^3 + 5y^2$

$\boxed{y' = \dfrac{-2xy^3 + 5y^2}{3x^2y^2 - 10xy - 4}}$

f) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

$\dfrac{2x}{9} + \dfrac{2y}{4}y' = 0$

$\dfrac{y}{2}y' = -\dfrac{2x}{9}$

$y' = -\dfrac{4x}{9y}$

4. Find the equation of the tangent line at the indicated point

a) $(y - x)^2 + y^3 = xy + 7$ at $(1, 2)$

$2(y-x)'(y'-1) + 3y^2\cdot y' = xy' + y$

$2(2-1)(y'-1) + 3(2)^2\cdot y' = 1y' + 2$

$2y' - 2 + 12y' = y' + 2$

$14y' - y' = 4$

$13y' = 4 \quad y' = \dfrac{4}{13}$

$y - 2 = \dfrac{4}{13}(x-1)$

b) $\dfrac{x^2}{16} + y^2 = 1$ at $\left(2, \frac{\sqrt{3}}{2}\right)$

$\dfrac{2x}{16} + 2y\cdot y' = 0$

$\dfrac{2(2)}{16} + \dfrac{2\sqrt{3}}{2}y' = 0$

$\sqrt{3}y' = -\dfrac{1}{4}$

$y' = \dfrac{-1}{4\sqrt{3}}$

$y - \dfrac{\sqrt{3}}{2} = \dfrac{-1}{4\sqrt{3}}($

$y - \dfrac{\sqrt{3}}{2} = \dfrac{-1}{4\sqrt{3}}(x-2)$ Final

5. Find the equation of the normal line at the indicated point

a) $y^3x + 2y = x^2$ at $(2, 1)$

$3y^2\cdot y'x + y^3 + 2y' = 2x$

$3\cdot y'(2) + 1 + 2y' = 4$

$6y' + 2y' = 3$

$8y' = 3$

$y' = \dfrac{3}{8} \perp -\dfrac{8}{3}$

$y - 1 = -\dfrac{8}{3}(x-2)$

b) $y\sqrt{x} - x\sqrt{y} = 12$ at $(9, 16)$

$y\cdot\dfrac{1}{2\sqrt{x}} + y'\sqrt{x} - x\dfrac{1}{2\sqrt{y}}\cdot y' - \sqrt{y} = 0$

$16\cdot\dfrac{1}{6} + 3y' - \dfrac{9}{8}y' - 4 = 0$

$\dfrac{15}{8}y' = \dfrac{4}{3} \quad y' = \dfrac{32}{45} \perp -\dfrac{45}{32}$

$y - 16 = \dfrac{-45}{32}(x-9)$

6. Find the points where the graph of $25x^2 + 16y^2 + 200x - 160y + 400 = 0$ has a horizontal and vertical tangent lines.

$25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$50x + 32y\cdot y' + 200 - 160y' = 0$

$32y\cdot y' - 160y' = -50x - 200$

$y' = \dfrac{-50x - 200}{32y - 160}$

$y' = \dfrac{25x + 100}{80 - 16y} = 0$ ✓ Hori.

→ is uno (vert) if $y = 5$

if $x = -4$

$25(-4)^2 + 16y^2 + 200(-4) - 160y + 400 =$

$16y^2 - 160y = 0$

$16y(y - 10) = 0$

$y = 0 \quad y = 10$

H.T → $(-4, 0)$ $(-4, 10)$

$25x^2 + 16(25) + 200x - 160(5) + 400$

$25x^2 + 200x = 0$

$25x(x + 8) = 0$

$x = 0, -8$

V.T → $(0, 5)$ $(-8, 5)$