AB Calculus Derivatives of Trig Functions Homework

Name: Key

1. Find dy/dx.

a) $y = 1 + x - \cos(x)$

$\frac{dy}{dx} = 1 + \sin x$

b) $y = \frac{1}{x} + 5\sin(x)$

$y' = -\frac{1}{x^2} + 5\cos x$

c) $y = 4 - x^2\sin(x)$

$y' = -x^2\cos x - 2x\sin x$

d) $y = \frac{4}{\cos(x)}$

$y' = \frac{\cos x(0) - 4(-\sin x)}{\cos^2 x} = \frac{4\sin x}{\cos^2 x}$

e) $y = \frac{\cot(x)}{1 + \cot(x)}$

$\frac{dy}{dx} = \frac{(1+\cot x)(-\csc^2 x) - (\cot x)(-\csc^2 x)}{(1+\cot x)^2}$

f) $y = 3x + x\tan(x)$

$y' = 3 + x\sec^2 x + \tan x$

2. Find an equation of the line tangent to the graph of $y = \sin(x) + 3$ at $x = \pi$.

$y' = \cos x$

$y' = \cos\pi = -1$

$y = \sin\pi + 3 = 3$

$y - 3 = -1(x - \pi)$

3. Find an equation of the line tangent to the graph of $y = x^2\cos(x)$ at $x = 3$.

$y = 9\cos 3 \approx -8.91$

$y' = 2x\cos x + x^2(-\sin x)$

$y' = 6\cos 3 - 9\sin 3 \approx -7.21$

$y + 8.91 = -7.21(x - 3)$

4. Assuming that $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$, prove each of the following.

a) $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}\frac{\cos x}{\sin x} = \frac{\sin x(-\sin x) - \cos x \cdot \cos x}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$

b) $\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}\frac{1}{\sin x}$

$\frac{\sin x(0) - 1(\cos x)}{\sin^2 x}$

$\frac{-\cos x}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cdot \cot x$

RHS ✓

5. Show that the graphs of $y = \sec(x)$ and $y = \cos(x)$ have horizontal tangents at $x = 0$.

$y' = \sec x \tan x \overset{?}{=} 0$ @ $x=0$ yes b/c $\tan 0 = 0$

$y' = -\sin x \overset{?}{=} 0$ @ $x=0$ yes b/c $\sin 0 = 0$

6. Find the equations for the lines that are tangent and normal to the curve $y = \sqrt{2}\cos(x)$ at the point $\left(\frac{\pi}{4}, 1\right)$.

$y' = -\sqrt{2}\sin x \quad y' = -\sqrt{2}\sin\frac{\pi}{4} = -\sqrt{2}\cdot\frac{\sqrt{2}}{2} = -1$

T: $y - 1 = -1\left(x - \frac{\pi}{4}\right)$ N: $y - 1 = 1\left(x - \frac{\pi}{4}\right)$

7. Find an equation for the tangent to the curve $y = 4 + \cot(x) - 2\csc(x)$ at $x = \frac{\pi}{2}$.

$y = 4 + \cot\frac{\pi}{2} - 2\csc\frac{\pi}{2}$

$4 + 0 - 2 = 2$

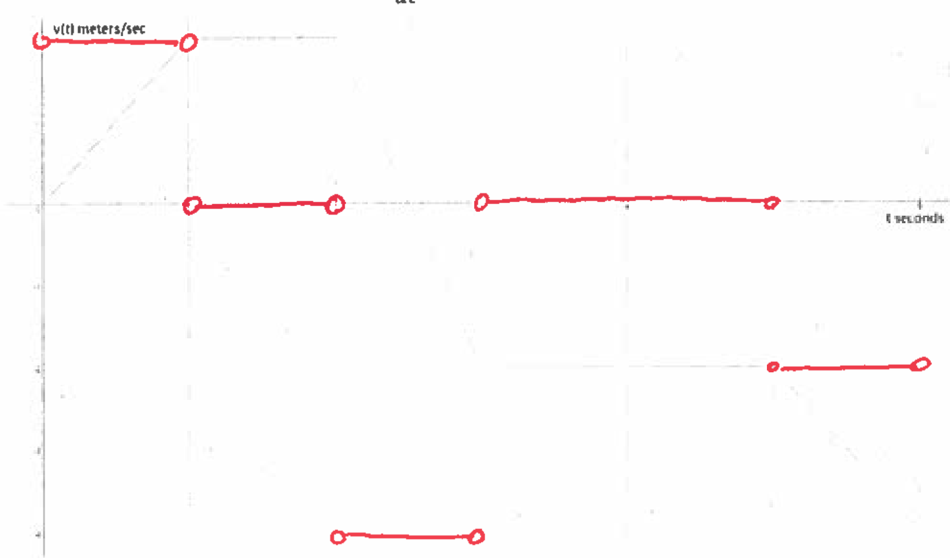
$y' = -\csc^2 x + 2\csc x \cot x$

$y' = -\csc^2\left(\frac{\pi}{2}\right) + 2\csc\left(\frac{\pi}{2}\right)\cot\left(\frac{\pi}{2}\right)$

$= -1 + 2(0) = -1$

$y - 2 = -1\left(x - \frac{\pi}{2}\right)$

8. The accompanying figure shows the velocity $v = \frac{ds}{dt} = f(t)$ meters/sec of a body moving along a coordinate line.



a) When does the particle moving left? Moving right? Standing still?

L: $(2.5, 6)$ R: $(0, 2.5)$ ↳ still only @ $x = 0$, $x = 2.5$ for an instant

b) Graph the particle's acceleration as a function of time t.

✓

c) When is the particle speeding up? Slowing down? Justify your answer.

Speed up $(0,1) \cup (2.5, 3) \cup (5, 6)$ Slow down $(2, 2.5)$

d) When is the particle's speed the greatest? Justify your answer.

$t = 6$ $v(t) = -4$ So speed $= |-4| = 4$