

Derivatives Practice

Use the function  $g(x) = 2x^3 - 3x + 5$  to answer questions 1 - 3

$$g(x+h) = 2(x+h)^3 - 3(x+h) + 5$$

1) Use the original limit definition of the derivative to find  $g'(x)$

$$g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

$$2(x^3 + 3x^2h + 3xh^2 + h^3) - 3x - 3h + 5$$

$$\frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 3x - 3h + 5 - 2x^3 + 3x - 5}{h}$$

$$\lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2) - 3 = 6x^2 - 3$$

$$g'(x) = 6x^2 - 3$$

2) Find the equation of a tangent line at  $x = 3$

$$g'(3) = 6(3)^2 - 3 = 54 - 3 = 51$$

$$g(3) = 2(3)^3 - 3(3) + 5 = 54 - 9 + 5 = 50$$

$$y - 50 = 51(x - 3)$$

3) Find the equation of a normal line at  $x = 3$

$$y - 50 = -\frac{1}{51}(x - 3)$$

4) If  $y = \frac{4}{3x^2}$  use the limit definition of the derivative to find  $\frac{dy}{dx}$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\frac{4}{3(x+h)^2} - \frac{4}{3x^2}}{h}$$

$$\frac{4x^2 - 4(x+h)^2}{3x^2(x+h)^2} = \frac{4x^2 - 4(x^2 + 2xh + h^2)}{3x^2(x+h)^2} = \frac{4x^2 - 4x^2 - 8xh - 4h^2}{3x^2(x+h)^2} = \frac{-8xh - 4h^2}{3x^2(x+h)^2}$$

$$\lim_{h \rightarrow 0} \frac{-8x - 4h}{3x^2(x+h)^2} = \frac{-8x}{3x^2 \cdot x^2} = \frac{-8}{3x^3}$$

$$\frac{2\sqrt{5}}{8} = \frac{\sqrt{5}}{4} = \frac{-8}{3x^3} = \frac{-8}{3 \cdot 4^3}$$

5) If  $f(x) = \sqrt{5x}$  use the alternative definition of the derivative to find  $f'(4)$

A)  $\frac{5\sqrt{20}}{20}$

B)  $\frac{\sqrt{2}}{5}$

C)  $\frac{\sqrt{5}}{4}$

D)  $\frac{\sqrt{5}}{20}$

E)  $\frac{1}{2\sqrt{2}}$

$$\lim_{x \rightarrow 4} \frac{\sqrt{5x} - \sqrt{20}}{x - 4} \cdot \frac{\sqrt{5x} + \sqrt{20}}{\sqrt{5x} + \sqrt{20}} \rightarrow \lim_{x \rightarrow 4} \frac{5x - 20}{(x-4)(\sqrt{5x} + \sqrt{20})} = \frac{5}{2\sqrt{20}}$$