AB Calculus Name________________________________

Derivatives Practice

Date________________ Period____

Use the function $g(x) = 2x^3 - 3x + 5$ to answer questions 1 – 3

$g(3) = 2(3)^3 - 3(3) + 5$

1) Use the original limit definition of the derivative to find $g'(x)$

$g(x+h) = 2(x+h)^3 - 3(x+h) + 5$

$2(x^3 + 3x^2h + 3xh^2 + h^3)$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2x^3} + 6x^2\cancel{h} + 6xh^{\cancel{2}} + 2h^{\cancel{3}2} - \cancel{3x} - 3\cancel{h} + \cancel{5} - \cancel{2x^3} + \cancel{3x} - \cancel{5}}{\cancel{h}}$$

$$\lim_{h \to 0} 6x^2 + 6xh + 2h^2 - 3 \to \boxed{g'(x) = 6x^2 - 3}$$

2) Find the equation of a tangent line at $x = 3$

$x, g(x), g'(x)$

$x = 3, g(3) = 50, g'(3) = 51$

$\boxed{y - 50 = 51(x - 3)}$

3) Find the equation of a normal line at $x = 3$

$\perp$

$\boxed{y - 50 = -\frac{1}{51}(x - 3)}$

4) If $y = \frac{4}{3x^2}$ use the limit definition of the derivative to find $\frac{dy}{dx}$

$$\lim_{h \to 0} \frac{\frac{x^2}{x^2} \cdot \frac{4}{3(x+h)^2} - \frac{4}{3x^2} \cdot \frac{(x+h)^2}{(x+h)^2}}{h}$$

shortcut: $y = \frac{4}{3}x^{-2} \xrightarrow{d/dx} \frac{dy}{dx} = -\frac{8}{3}x^{-3}$

$$\lim_{h \to 0} \frac{4x^2 - 4(x^2 + 2xh + h^2)}{3x^2(x+h)^2 h} \to \lim_{h \to 0} \frac{\cancel{4x^2} - \cancel{4x^2} - 8x\cancel{h} - 4h^{\cancel{2}}}{3x^2(x+h)^2 \cancel{h}}$$

$$\lim_{h \to 0} \frac{-8x - 4h}{3x^2(x+h)^2} \to \frac{-8x}{3x^4} \to \boxed{\frac{-8}{3x^3} = \frac{dy}{dx}}$$

$f(4) = \sqrt{20}$

5) If $f(x) = \sqrt{5x}$ use the alternative definition of the derivative to find $f'(4)$

A) $\frac{5\sqrt{20}}{20}$ B) $\frac{\sqrt{2}}{5}$ C) $\frac{\sqrt{5}}{4}$ D) $\frac{\sqrt{5}}{20}$ E) $\frac{1}{2\sqrt{2}}$

$$\lim_{x \to 4} \frac{(\sqrt{5x} - \sqrt{20})}{(x-4)} \cdot \frac{(\sqrt{5x} + \sqrt{20})}{(\sqrt{5x} + \sqrt{20})} \to \lim_{x \to 4} \frac{5x - 20 \to 5(\cancel{x-4})}{(\cancel{x-4})(\sqrt{5x} + \sqrt{20})} \to \frac{5}{\sqrt{20} + \sqrt{20}} = \frac{5}{2\sqrt{20}}$$

$$\frac{5}{2\sqrt{4}\sqrt{5}} \to \frac{5}{2 \cdot 2\sqrt{5}} \to \frac{5}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \to \frac{\cancel{5}\sqrt{5}}{4 \cdot \cancel{5}}$$