4

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ conjugate way

What is the value of $\lim_{h\to 0}\frac{(8+h)^{\frac{1}{3}}-2}{h}$? $f'(8)=$

$\cancel{\frac{(\sqrt[3]{8+h}-2)\sqrt[3]{8+h}}{h}}$

(A) 0

(B) $\frac{1}{12}$

(C) $\frac{1}{4}$

(D) 1

$f(x)=\sqrt[3]{x}$ at $x=8$

$f(x)=x^{\frac{1}{3}}$

$f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$

$=\frac{1}{3\sqrt[3]{x^2}} \to \frac{1}{3\sqrt[3]{8^2}}$

5

Let f and g be differentiable functions such that $f'(1)=2$ and $g'(1)=6$. If $h(x)=5f(x)-4g(x)+3x^2-2$, what is the value of $h'(1)$?

(A) -10

(B) -8

(C) 2

(D) 40

$h'(x)=5f'(x)-4g'(x)+6x$

$h'(1)=5f'(1)-4g'(1)+6(1)$

$5(2)-4(6)+6$

$10-24+6$

$16-24$

(6)

If $f(x) = x^3 - x^2 + x - 1$, then $f'(2) =$

(A) 10

(B) 9

(C) 7

(D) 5

(E) 3

$f'(x) = 3x^2 - 2x + 1$

$f'(2) = 3(2)^2 - 2(2) + 1$

$12 - 4 + 1$

$13 - 4$

(7)

If $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$, then $f'(4) =$

(A) $\frac{1}{16}$

(B) $\frac{5}{16}$

(C) 1

(D) $\frac{7}{2}$

(E) $\frac{49}{4}$

$f(x) = x^{\frac{1}{2}} + 3x^{\frac{-1}{2}}$

$f'(x) = \frac{1}{2}x^{\frac{-1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2\sqrt{x}} - \frac{3}{2\sqrt{x^3}}$

$= \frac{1}{2\sqrt{4}} - \frac{3}{2\sqrt{4^3}} = \frac{1}{4} - \frac{3}{16}$

$\frac{4}{16} - \frac{3}{16}$

(8)

If f is the function given by $f(x) = \frac{4}{x} + 5x - 1$, then $f'(2) =$

$\frac{4}{x} = 4\frac{1}{x}$

(A) 4

(B) 6

(C) 7

(D) 11

$f(x) = 4x^{-1} + 5x - 1$

$f'(x) = -4x^{-2} + 5$

$= \frac{-4}{x^2} + 5$

$\frac{-4}{4} + 5$

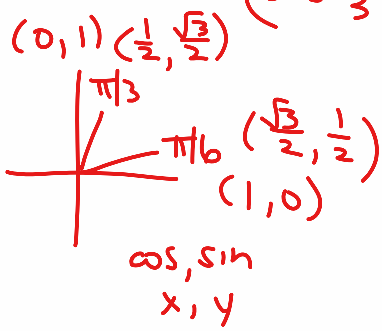
14. $\frac{d}{dx}$ If y = sin x cos x, then at $x = \frac{\pi}{3}$, $\frac{dy}{dx} =$

d 1st 2nd d 2nd 1st

- (A) $-\frac{1}{2}$
- (B) $-\frac{1}{4}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{2}$
- (E) 1

$$\frac{dy}{dx} = (\cos x)(\cos x) + (-\sin x)(\sin x)$$

$$\left(\cos\frac{\pi}{3}\right)\left(\cos\frac{\pi}{3}\right) + \left(-\sin\frac{\pi}{3}\right)\left(\sin\frac{\pi}{3}\right)$$



$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{4} - \frac{3}{4} = -\frac{2}{4}$$

---

15. If $f(x) = \frac{3x-2}{2x+3}$, then f'(x)=

Quotient Rule

- (A) $-\frac{13}{(2x+3)^2}$
- (B) $\frac{3}{(2x+3)^2}$
- (C) $\frac{5}{(2x+3)^2}$
- (D) $\frac{13}{(2x+3)^2}$
- (E) $\frac{12x+5}{(2x+3)^2}$

$$\frac{(2x+3)(3) - (3x-2)(2)}{(2x+3)^2}$$

$$6x + 9 - 6x + 4$$

---

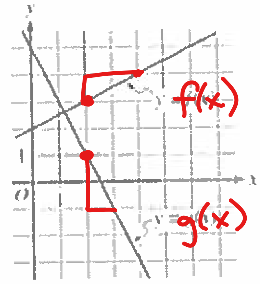
16. Which of the following correctly shows the derivation of $\frac{d}{dx}(\sec x)$?

- (A) $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{1}{\frac{d}{dx}(\cos x)} = \frac{1}{\sin x}$
- (B) $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{1}{\frac{d}{dx}(\cos x)} = \frac{1}{-\sin x}$
- (C) $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{\frac{d}{dx}(1)\cdot\cos x - 1\cdot\frac{d}{dx}(\cos x)}{(\cos x)^2} = \frac{0\cdot\cos x - 1\cdot(-\sin x)}{(\cos x)^2}$
- (D) $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{\frac{d}{dx}(1)\cdot\cos x + 1\cdot\frac{d}{dx}(\cos x)}{(\cos x)^2} = \frac{0\cdot\cos x + 1\cdot(-\sin x)}{(\cos x)^2}$

tanx · secx

$$\frac{\sin x}{\cos x} \quad \frac{1}{\cos x}$$

(17)



The figure above shows the graphs of the functions f and g. If $h(x) = f(x)g(x)$, then $h'(2) =$

$h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

$h'(2) = f'(2)\cdot g(2) + f(2)\,g'(2)$

$= (\frac{1}{2})(1) + (3)(-2)$

$\frac{1}{2} - 6 = -5\frac{1}{2}$

(A) $\frac{13}{2}$

(B) $\frac{1}{2}$

(C) -1

(D) $-\frac{11}{2}$

(18)

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
−1	2	5	−2

The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

$$\frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$$

$$\frac{(2)(5) - (-1)(-2)}{2^2}$$

$\frac{10-2}{4} = \frac{8}{4}$

(A) $-\frac{5}{2}$

(B) -2

(C) 2

(D) 3

(E) 8

22. If $y = \frac{1}{2}x^{4/5} - \frac{3}{x^5}$, then $\frac{dy}{dx} =$

$$\frac{d}{dx}\left(\frac{1}{2}x^{\frac{4}{5}} - 3x^{-5}\right)$$

$$\frac{4}{10}x^{\frac{-1}{5}} + 15x^{-6}$$

$$\frac{2}{5x^{\frac{1}{5}}} + \frac{15}{x^6}$$

(A) $\frac{2}{5x^{1/5}} + \frac{15}{x^6}$

(B) $\frac{2}{5x^{1/5}} + \frac{15}{x^4}$

(C) $\frac{2}{5x^{1/5}} - \frac{3}{5x^4}$

(D) $\frac{2x^{1/5}}{5} + \frac{15}{x^6}$

(E) $\frac{2x^{1/5}}{5} - \frac{3}{5x^4}$

---

23. If $y = x\sin x$, then $\frac{dy}{dx} =$

$$\frac{dy}{dx} = 1 \sin x + x \cos x$$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

---

24. If $y = 5x\sqrt{x^2+1}$, then $\frac{dy}{dx}$ at $x = 3$ is

$$(5x)(x^2+1)^{\frac{1}{2}}$$

$$(5x)\ \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x) + (5)(x^2+1)^{\frac{1}{2}}$$

$$\frac{15\cdot(3)}{\sqrt{10}} + 5\sqrt{10}$$

(A) $\frac{5}{2\sqrt{10}}$

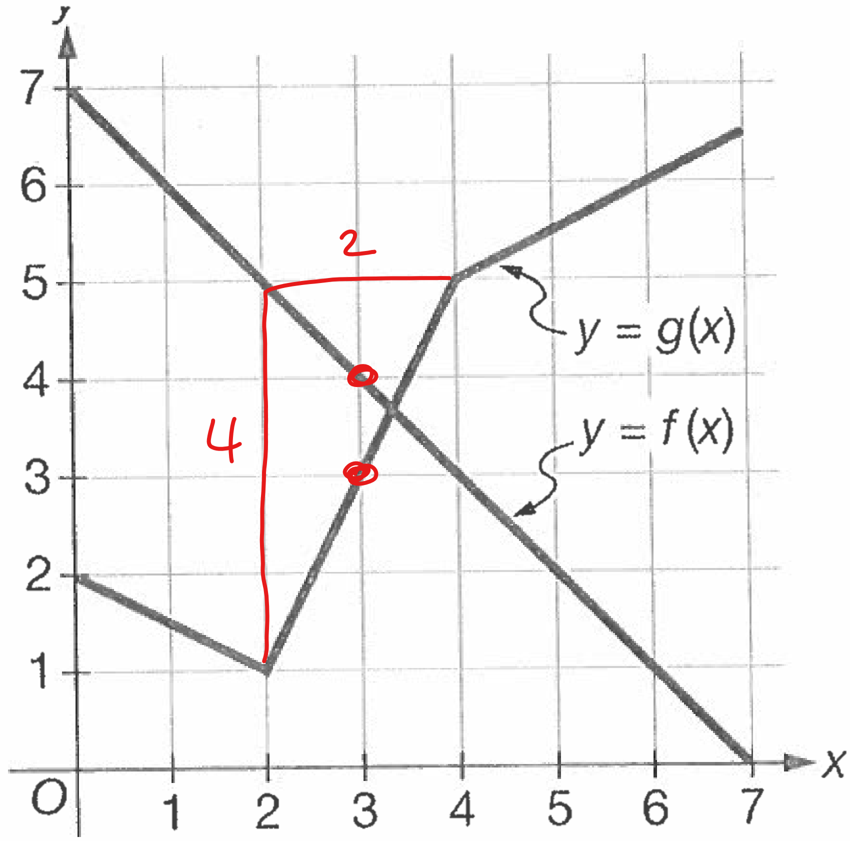
(B) $\frac{15}{\sqrt{10}}$

(C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$

(D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$

(E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

25



The graphs of the linear function f and the piecewise linear function g are shown in the figure above. If $h(x) = f(x)\,g(x)$, then $h'(3) =$

(A) -11

(B) -2

(C) 2

(D) 5

$h'(x) = f'(x)\,g(x) + f(x) \cdot g'(x)$

$h'(3) = f'(3)\,g(3) + f(3)\,g'(3)$

$(-1)(3) + (4)\left(\frac{4}{2}\right)$

$-3 + 8$

26

The velocity v, in meters per second, of a certain type of wave is given by $v(h) = 3\sqrt{h}$, where h is the depth, in meters, of the water through which the wave moves. What is the rate of change, in meters per second per meter, of the velocity of the wave with respect to the depth of the water, when the depth is 2 meters?

$v(h)$ = units of what = m/s

h = units of what = m

(A) $-\frac{3}{4\sqrt{2}}$

(B) $-\frac{3}{8\sqrt{2}}$

(C) $\frac{3}{2\sqrt{2}}$

(D) $\frac{3}{\sqrt{2}}$

(E) $4\sqrt{2}$

$\frac{3}{2\sqrt{2}}$ $\quad v'(h) \leftrightarrow \frac{d}{dh} v(h)$

$v(h) = 3h^{\frac{1}{2}}$

$v'(h) = \frac{3}{2} h^{-\frac{1}{2}} = \frac{3}{2\sqrt{h}}$