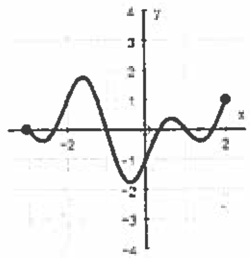
AB Calculus Differentiability Homework Name: Key

1. When does a derivative fail to exist?

cusp, vertical tangent, sharp turn, not continuous

2. For each of the following, use the graph to find the x-values with the given characteristic over the given domain.

I. $-3 \le x \le 2$



a) Find the x-values where the graph is differentiable over the domain.

$(-3, 2)$

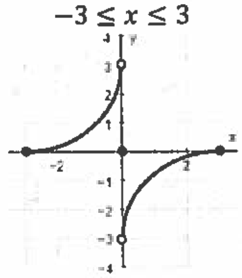
b) Find the x-values where the graph is continuous but not differentiable.

none

c) Find the x-values where the graph is neither continuous nor differentiable.

none

II. $-3 \le x \le 3$



a) Find the x-values where the graph is differentiable over the domain.

$(-3, 0) \cup (0, 3)$

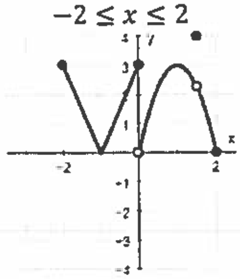
b) Find the x-values where the graph is continuous but not differentiable.

none

c) Find the x-values where the graph is neither continuous nor differentiable.

$x = 0$

III. $-2 \le x \le 2$



a) Find the x-values where the graph is differentiable over the domain.

$(-2, -1) \cup (-1, 0) \cup (0, 1.5) \cup (1.5, 2)$

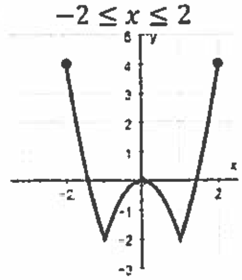
b) Find the x-values where the graph is continuous but not differentiable.

$x = -1$

c) Find the x-values where the graph is neither continuous nor differentiable.

$x = 0, 1.5$

IV. $-2 \le x \le 2$



a) Find the x-values where the graph is differentiable over the domain.

$(-2, -1) \cup (-1, 1) \cup (1, 2)$

b) Find the x-values where the graph is continuous but not differentiable.

$x = -1, 1$

c) Find the x-values where the graph is neither continuous nor differentiable.

none

3. (Calculator) Find the equation of the tangent line to the graph of $f(x) = x^3 + x^2$ at $x = 2$.

Math $\rightarrow$ 8 (nDeriv) $\rightarrow x^3 + x^2 \big|_{x=2} \rightarrow f'(2) = 16$

$f(2) = 2^3 + 2^2 = 8 + 4 = 12$

$\boxed{y - 12 = 16(x - 2)}$

4. (Calculator) Determine if $g(x)$ defined below is differentiable at $x = 0$. Justify your response.

$$g(x) = \begin{cases} 3x - 2, & x \leq 0 \\ x^2 - 1, & x > 0 \end{cases}$$

① Continuous? $3(0) - 2 = (0)^2 - 1$?

$-2 \neq -1$ Not continuous so not differentiable

5. Let f be a function that is differentiable on the open interval $(0, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, each of the following statements must be true. Explain why each statement is true.

implies continuity

a) f has at least two zeros.

↳ True. By IVT. since f is continuous on closed interval $[2, 9]$ and $-5 < 0 < 5$, there is a c in $(2, 5)$ and another in $(5, 9)$ such that $f(c) = 0$.

b) The graph of f has at least one horizontal tangent line.

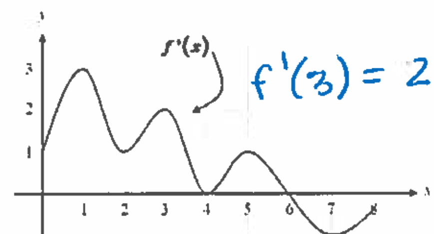
since the graph is differentiable, it is also continuous and since it must increase and then decrease, there must be a maximum point where the tangent line is horizontal.

c) For some c inside the interval $2 < c < 5$, $f(c) = 3$.

$f(2) = -5$
$f(5) = 5$

$f(x)$ is continuous on closed interval $[2, 5]$ and $-5 < 3 < 5$, so by IVT, there is a c in $(2, 5)$ such that $f(c) = 3$.

6. The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown below. The point $(3, 5)$ is on the graph of $f(x)$. Find the equation of the tangent line to the graph of f at $(3, 5)$.



$f'(3) = 2$

$y - y_1 = m(x - x_1)$

$y - 5 = 2(x - 3)$

7. Use the limit definition of derivative to find $f'(x)$ if $f(x) = 3x^2 - 5$.

$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} \to \lim\limits_{h \to 0} \dfrac{3(x+h)^2 - 5 - (3x^2 - 5)}{h} \to \dfrac{3(x^2 + 2xh + h^2) - 5 - 3x^2 + 5}{h}$

$\to \dfrac{3x^2 + 6xh + 3h^2 - 5 - 3x^2 + 5}{h} \to \dfrac{6xh + 3h^2}{h} \to 6x + 3h \to 6x$ $\boxed{f'(x) = 6x}$

8. Use the alternative definition of derivative to find $f'(x)$ if $f(x) = \frac{5}{3x+2}$ at $x = 1$. $f(1) = \dfrac{5}{3(1)+2} \to \dfrac{5}{5} = 1$

$\lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c} \to \lim\limits_{x \to 1} \dfrac{\frac{5}{3x+2} - 1}{x - 1}$

$\to \dfrac{\frac{5}{3x+2} - \frac{3x+2}{3x+2}}{x-1} \to \dfrac{5 - 3x - 2}{(x-1)(3x+2)} \to \dfrac{-3x + 3}{(x-1)(3x+2)} \to \dfrac{-3(x-1)}{(x-1)(3x+2)} \to \dfrac{-3}{3x+2}$

$\to \dfrac{-3}{3(1)+2} \to \dfrac{-3}{5} \to$ $\boxed{f'(1) = -3/5}$