AB Calculus Diff Eq. All the Things Practice Name: Key

1. Evaluate each of the following integrals.

a) $\int \left(5x^2 + 2^x - \sqrt[3]{x^4} + \frac{1}{x}\right) dx$

b) $\int_1^4 \left(\frac{1}{x^3} - \sqrt{x}\right) dx$

c) $\int \sqrt{2x-3}\, dx$

d) $\int \cos^3 x \sin x\, dx$

e) $\int \tan x\, dx$

f) $\int_1^2 \frac{2\ln x}{x} dx$

g) $\int \frac{1}{16+9x^2} dx$

h) $\int \frac{7}{\sqrt{25-4x^2}} dx$

2. Find the general and particular solutions for the following differential equations with the given initial condition.

a) $\frac{dy}{dx} = \frac{4x}{y}$ and $y(0) = 5$

b) $\frac{dy}{dx} = 3x^2y$ and $y(0) = 2$

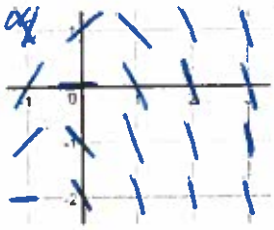
c) $\frac{dy}{dx} = 4xy^2$ and $y(0) = 1$

d) $\frac{dy}{dx} = \frac{y^2}{x}$ and $y(1) = \frac{1}{3}$

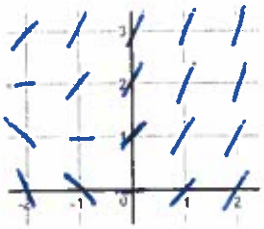
3. Find the equations for the velocity and position of a particle if the acceleration of a particle is given by $a(t) = -32\ ft/sec^2$, and, at time $t = 0$, the velocity of a particle is 64 ft/sec and the position is 32 ft.

4. On the axes provided, sketch a slope field for each differential equation.

a) $\frac{dy}{dx} = y - 2x$



b) $\frac{dy}{dx} = x + y$



5. Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$, find an approximation for $y(1)$ by using Euler's method with two equal steps.

$y(1) \approx 5.25$

6. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that $f(2) = 5$, use Euler's method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5	5.2	5.5

$f(3) \approx 5.5$

1.) $\int (5x^2 + 2^x - \sqrt[3]{x^4} + \frac{1}{x})\,dx$

(a)

$\int (5x^2 + 2^x - x^{4/3} + x^{-1})\,dx$

$= \frac{5x^3}{3} + \frac{2^x}{\ln 2} - \frac{3x^{7/3}}{7} + \ln|x| + C$

(b) $\int_1^4 (x^{-3} - x^{\frac{1}{2}})\,dx$

$= \frac{x^{-2}}{-2} - \frac{2x^{\frac{3}{2}}}{3}\Big|_1^4$

$\left(\frac{-1}{2}(4)^{-2} - \frac{2}{3}(4)^{\frac{3}{2}}\right) - \left(\frac{-1}{2}(1)^{-2} - \frac{2}{3}(1)^{\frac{3}{2}}\right)$

$= \left(\left(\frac{-1}{2}\right)\left(\frac{1}{16}\right) - \frac{2}{3}(8)\right) - \left(\frac{-1}{2} - \frac{2}{3}\right)$

$\Rightarrow$ calculator fine $\rightarrow$ $-403/96$

(c) $\frac{1}{2}\int \sqrt{u}\,du$ $\quad u = 2x - 3$

$\frac{1}{2}\int u^{\frac{1}{2}}\,du$ $\quad \frac{du}{dx} = 2$

$\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$ $\quad \frac{1}{2}du = dx$

$\frac{1}{3}u^{\frac{3}{2}} + C \rightarrow \frac{1}{3}(2x-3)^{\frac{3}{2}} + C$

(d) $\int \cos^3 x \sin x\,dx$ $\quad u = \cos x$

$-\int u^3\,du$ $\quad \frac{du}{dx} = -\sin x$

$-\frac{1}{4}u^4 + C \rightarrow -\frac{1}{4}\cos^4 x + C$ $\quad -du = \sin x\,dx$

(e) $\int \tan x\,dx \rightarrow \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$ $\quad \rightarrow \frac{1}{-1}\int \frac{du}{u}$

$\frac{du}{dx} = -\sin x$ $\quad \rightarrow -1\ln|u| + C$

$\frac{du}{-1} = \sin x\,dx$ $\quad \rightarrow -\ln|\cos x| + C$

(f) $\int_1^2 \frac{2\ln x}{x}\,dx$ $\quad u = \ln x$ $\quad \frac{du}{dx} = \frac{1}{x}$

$2\int_0^{\ln 2} u\,du$ $\quad u = \ln 2$, $u = \ln 1 = 0$

$\rightarrow 2\frac{u^2}{2}\Big|_0^{\ln 2} \rightarrow (\ln 2)^2 - 0^2$

$= (\ln 2)^2$

(g) $\int \frac{1}{16 + 9x^2}\,dx \rightarrow \int \frac{1}{16\left(1 + \frac{9}{16}x^2\right)}\,dx$

$\rightarrow \frac{1}{16}\int \frac{1}{1 + \left(\frac{3}{4}x\right)^2}\,dx$ $\quad \rightarrow u = \frac{3}{4}x$

$\frac{du}{dx} = \frac{3}{4}$

$\rightarrow \frac{1}{16}\cdot\frac{84}{63}\int \frac{1}{1+u^2}\,du$ $\quad du = \frac{3}{4}dx$

$\frac{4}{3}du = dx$

$\rightarrow \frac{1}{12}\tan^{-1}u + C \rightarrow \frac{1}{12}\tan^{-1}\left(\frac{3}{4}x\right) + C$

(h) $\int \frac{7}{\sqrt{25 - 4x^2}}\,dx$ $\quad u = \frac{2}{5}x$

$\int \frac{7}{\sqrt{25\left(1 - \left(\frac{2}{5}x\right)^2\right)}}\,dx$ $\quad \frac{du}{dx} = \frac{2}{5}$

$\frac{7}{5}\int \frac{8\,1}{\sqrt{1 - \left(\frac{2}{5}x\right)^2}}\,dx$ $\quad \frac{5}{2}du = dx$

$\frac{7}{5}\cdot\frac{5}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\rightarrow \frac{7}{2}\sin^{-1}u + C \rightarrow \frac{7}{2}\sin^{-1}\left(\frac{2}{5}x\right) + C$

2

(a) $\frac{dy}{dx} = \frac{4x}{y}$

$\int y\,dy = \int 4x\,dx$

$\frac{y^2}{2} = 2x^2 + C \rightarrow (0,5)$

$\frac{25}{2} = 0 + C \rightarrow C = \frac{25}{2}$

$\frac{y^2}{2} = \frac{2x^2 + 25}{2}$

$y^2 = 4x^2 + 25$

$y = \sqrt{4x^2 + 25}$

↳ Be careful b/c sometimes it's negative depending on initial condition.

(b) $\frac{dy}{dx} = 3x^2 y$

$\int \frac{dy}{y} = \int 3x^2\,dx$

$\ln|y| = x^3 + C \rightarrow (0,2)$

$\ln 2 = C$

$\ln|y| = x^3 + \ln 2$

$y = e^{(x^3 + \ln 2)}$

$\rightarrow e^{x^3} \cdot e^{\ln 2}$

$y = 2e^{x^3}$

(c) $\frac{dy}{dx} = 4xy^2$

$\int \frac{dy}{y^2} = \int 4x\,dx$

$-\frac{1}{y} = 2x^2 + C \rightarrow (0,1)$

$-1 = 2(0)^2 + C \rightarrow C = -1$

$y \cdot \left(-\frac{1}{y} = 2x^2 - 1\right) \cdot y$

$-1 = (2x^2 - 1) \cdot y$

$y = \frac{-1}{2x^2 - 1}$

d) $\frac{dy}{dx} = \frac{y^2}{x}$

$\int \frac{dy}{y^2} = \int \frac{1}{x}\,dx$

$-\frac{1}{y} = \ln|x| + C$

$(1, \frac{1}{3})$ $\quad -\frac{1}{1/3} = \ln|1| + C$

$-3 = C$

$-\frac{1}{y} = \ln|x| - 3$

$\frac{-1}{\ln|x| - 3} = y \rightarrow \boxed{y = \frac{-1}{\ln|x| - 3}}$

e)

3) $a(t) = -32$

$v(0) = 64$

$s(0) = 32$

$\int a(t) = \int v'(t)$

$-32t + C = v(t) \rightarrow (0, 64) \rightarrow \boxed{v(t) = -32t + 64}$

$\int v(t) = \int s'(t)$

$-16t^2 + 64t + C = S(t) \rightarrow (0, 32) \rightarrow \boxed{S(t) = -16t^2 + 64t + 32}$

4.) on graphs

6)

old pt	dx	$m = \frac{dy}{dx}$	$dy = m \cdot dx$	new pt
(2, 5)	.5	.4	.2	(2.5, 5.2)
(2.5, 5.2)	.5	.6	.3	(3, 5.5)

5.)

old pt	dx	$m = dy/dx$	$dy = m \cdot dx$	new pt
(0, 3)	.5	2	2(.5) = 1	(.5, 4)
(.5, 4)	.5	2.5	2.5(.5) = 1.25	(1, 5.25)