AB Calculus Rules for Differentiation Day 1 Practice Name: Key

1. What does the derivative of a function tell you about the function?

Slope of the tangent line

2. What is the power rule for derivatives? (How do you take the derivative of $y = x^n$?

$y' = nx^{n-1}$

3. For each of the following, find $\frac{dy}{dx}$

39) w/o product rule $x^3+4x^2-3x-12$ $\frac{d}{dx}$ $3x^2+8x-3$

a) $y = -2x^3 + x$ $y' = -6x^2+1$

b) $y = \frac{x^4}{3} - \frac{x^2}{7} + 5$ $y' = \frac{4}{3}x^3 - \frac{2}{7}x$

c) $y = -\frac{5}{x^2} + \frac{6}{x} - 8x^3$ $-5x^{-2}+6x^{-1}-8x^3$ $y' = 10x^{-3} - 6x^{-2} - 24x^2$

d) $y = \frac{x^{-3}}{2} + 5x^{-4} - 3x^{-6}$ $y' = -\frac{3}{2}x^{-4} - 20x^{-5} + 18x^{-7}$

e) $y = 5x^4 + 2x^3 - 8x^2 - 7x + 11$ $y' = 20x^3 + 6x^2 - 16x - 7$

f) $y = 7x - 8$ $y' = 7$

g) $y = (x^2 - 3)(x + 4)$ $y' = (x^2-3)(1) + (2x)(x+4)$

$y = \sqrt{x} + \frac{3}{\sqrt{x}} - 6x^{\frac{5}{3}} + \frac{7}{x^3}$ $y = \frac{1}{2\sqrt{x}} - \frac{3}{2x\sqrt{x}} - 10x^{2/3} - 21x^{-4}$

i) $y = \frac{x^5 - 2x^4 + 3x^3}{x^5}$ $1 - 2x^{-1} + 3x^{-2}$ $y' = 2x^{-2} - 6x^{-3}$

4. (Calculator) Find all points where $y = x^4 - 5x^3 - 3x^2 + 13x + 10$ has a horizontal tangent line.

$y' = 4x^3 - 15x^2 - 6x + 13 = 0$ $x \approx 3.921, .829, -1$ ↳ exact value

5. Find the equation of the tangent line to the function $y = \frac{x^2+x-2}{2x}$ at the point where $x = 1$. $y = \frac{1^2+1-2}{2} = 0$

$y = \frac{1}{2}x + \frac{1}{2} - x^{-1} \rightarrow y' = \frac{1}{2} + x^{-2} \rightarrow y'(1) = \frac{1}{2} + 1^{-2} = \frac{3}{2}$ $y = \frac{3}{2}(x-1)$

6. Find the equation of the normal line to the function $y = x^3 - 5x + 1$ at the point where $x = 2$. $y = 8 - 10 + 1 = -1$

$y' = 3x^2 - 5 \rightarrow y'(2) = 3(2)^2 - 5 = 12 - 5 = 7 \rightarrow -1/7$ $y + 1 = -\frac{1}{7}(x-2)$

7. Find the points on the curve $y = x^3 + 3x^2 - 9x + 7$ where the tangent line is parallel to the x-axis.

$y' = 3x^2 + 6x - 9 = 0$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1)$ $x = -3, 1$ $(-3, 34)$ $(1, 2)$

8. Find all x-values on the curve $y = x^3 + x$ where the slope is 4.

$y' = 3x^2 + 1 = 4$ $3x^2 = 3$ $x^2 = 1$ $x = \pm 1$

9. The normal line to the graph of f at $(1, 2)$ passes through the point $(-1, 1)$. Find the value of $f'(1)$.

$\frac{1}{2}$ → slope of normal line Tangent line → -2 $f'(1) = -2$

10. Find the values of a and b so that $g(x)$ is both continuous and differentiable at $x = 0$.

$$g(x) = \begin{cases} ax + b, & x > 0 \\ 1 - x + x^2, & x \le 0 \end{cases}$$

$a(0) + b = 1 - 0 + 0$ $b = 1$ $a = -1$

$g'(x) = a \quad x > 0$ $= -1 + 2x \quad x \le 0 \rightarrow -1 + 0 = -1$

11. Let $f(x) = \sqrt{x}$. Find c if the rate of change of f at $x = c$ is twice the rate of change at $x = 1$.

$f'(x) = \frac{1}{2\sqrt{x}}$ $f'(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$ $f'(c) = 2 \cdot \frac{1}{2} = 1$ $\frac{1}{2\sqrt{x}} = 1 \rightarrow 1 = 2\sqrt{x}$ $\frac{1}{2} = \sqrt{x}$ $x = \frac{1}{4}$

12. Find the equation of the tangent line to the function $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$.

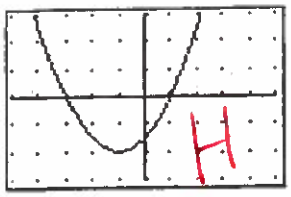
$f'(x) = 4x^3 + 4x = 1$ $f(.23\ldots) \approx .115$

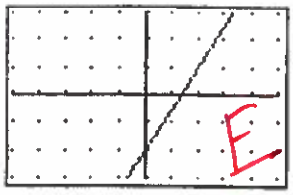
$4x^3 + 4x - 1 = 0$

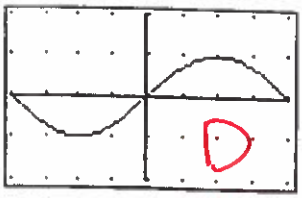
$x \approx .2367329$ $y - .115 = 1(x - .237)$

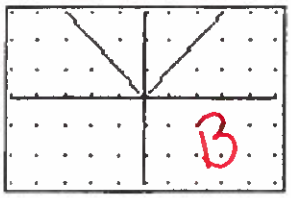
Match It! match the graph of a function to the graph of its derivative

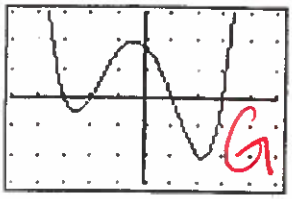
I. $y = f(x)$

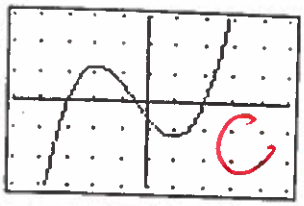
1. 

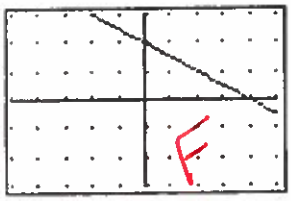
2. 

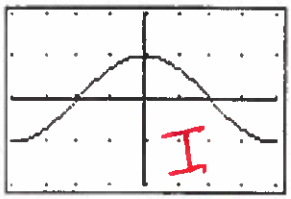
3. 

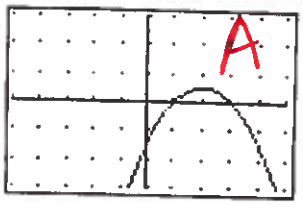
4. 

5. 

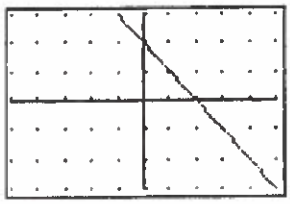
6. 

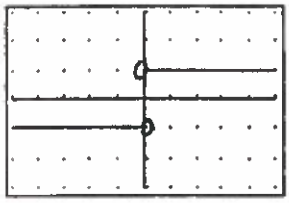
7. 

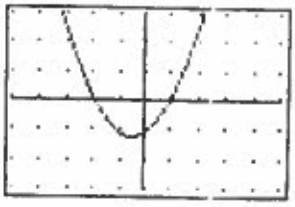
8. 

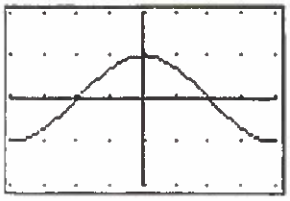
9. 

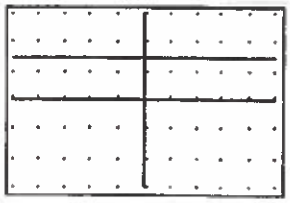
II. $y = f'(x)$

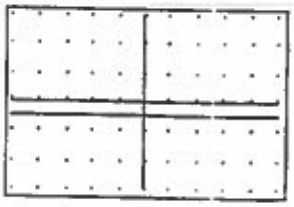
a. 

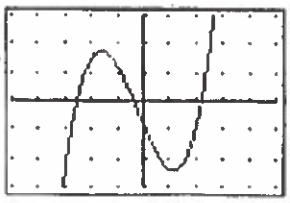
b. 

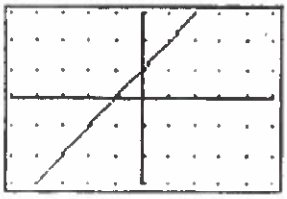
c. 

d. 

e. 

f. 

g. 

h. 

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