ABCALC Differentiation Review Session Problems

Key

Find the derivative of each function

1. $y = \frac{2}{3}x^4 + 5x - x^{-3}$

$\frac{dy}{dx} = \frac{8}{3}x^3 + 5 + \frac{3}{x^4}$

2. $y = \frac{2}{3}\sqrt[3]{x^2} - \frac{3}{4}x^{\frac{3}{5}}$ $\rightarrow \frac{2}{3}x^{\frac{2}{3}}$

$\frac{dy}{dx} = \frac{4}{9}x^{-\frac{1}{3}} - \frac{9}{20}x^{-\frac{2}{5}}$

Find the point where $f(x) = x^2 - 6x + 11$ has a horizontal tangent line.

$f'(x) = 2x - 6$

$0 = 2x - 6$

$x = 3$

$f(3) = 3^2 - 6(3) + 11$

$9 - 18 + 11$

$f(3) = 2$

$(3, 2)$

Find the derivative of each function

3) $y = (2x^4 - 3)(x^2 + 1)$

$y = 2x^6 + 2x^4 - 3x^2 - 3$

$\frac{dy}{dx} = 12x^5 + 8x^3 - 6x$

4) $y = \frac{4x^5 + x^2 + 4}{5x^2 - 2}$

$\frac{dy}{dx} = \frac{(5x^2 - 2)(20x^4 + 2x) - (4x^5 + x^2 + 4)(10x)}{(5x^2 - 2)^2}$

6. Evaluate $\lim_{h \to 0} \frac{(x+h)^8 - x^8}{h} \rightarrow \frac{dy}{dx}x^8 = 8x^7$

Free Response Practice 1 (no calculator)

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a)

$$(1)(y^2) + (x)(2y)\left(\frac{dy}{dx}\right) - \left(\left(x^3\frac{dy}{dx} + 3x^2y\right)\right) = 0$$

$$y^2 + 2xy\frac{dy}{dx} - x^3\frac{dy}{dx} - 3x^2y = 0$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b)

$$(1)y^2 - (1)y = 6$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3$$

$$y = -2$$

$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3} = 0$$

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3} = \frac{-10}{-5} = 2$$

$y - 3 = 0(x-1)$ and $y + 2 = 2(x-1)$

(c)

$$2xy - x^3 = 0$$

$$y = \frac{x^3}{2x} = \frac{x^2}{2}$$

$$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$$

$$\frac{x^5}{4} - \frac{x^5}{2} = 6$$

$$x^5 - 2x^5 = 24$$

$$-x^5 = 24$$

$$x^5 = -24$$

$$x = \sqrt[5]{-24}$$

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Find the derivative of each function

$(4x^2+2)^{\frac{1}{2}}$

$(\sin(x^3))^2$ $x^2 \to 2x$, $\sin x \to \cos x$, $x^3 \to 3x^2$

a) $f(x) = \sqrt{4x^2+2}$

$f'(x) = \frac{1}{2\sqrt{4x^2+2}}(8x)$

b) $f(x) = \sin^2(x^3)$

$f'(x) = 2\sin(x^3)\cdot\cos(x^3)\cdot 3x^2$

c) $f(x) = \frac{\sec(5x)}{2} \to \frac{1}{2}\sec(5x)$

$f'(x) = \frac{1}{2}\sec(5x)\tan(5x)\cdot 5$

d) $f(x) = \tan^2(\sqrt{x^3+5x})$

$x^2 \to 2x$

$\tan x \to \sec^2 x$

$\sqrt{x} \to \frac{1}{2\sqrt{x}}$

$x^3+5x \to 3x^2+5$

$f'(x) = 2\tan\sqrt{x^3+5x}\cdot\sec^2\sqrt{x^3+5x}\cdot\frac{1}{2\sqrt{x^3+5x}}\cdot(3x^2+5)$

Find the equation of the tangent line to $y=\sin^2 x$ at $x=\frac{\pi}{3}$

$y=(\sin x)^2$

$y(\frac{\pi}{3}) = (\sin\frac{\pi}{3})^2 = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$

$\frac{dy}{dx} = 2\sin x\cos x$

$\frac{dy}{dx}\Big|_{x=\frac{\pi}{3}} = 2\sin\frac{\pi}{3}\cos\frac{\pi}{3} = 2(\frac{\sqrt{3}}{2})(\frac{1}{2}) = \frac{\sqrt{3}}{2}$

$y-\frac{3}{4} = \frac{\sqrt{3}}{2}(x-\frac{\pi}{3})$

Find the equation of the tangent line at the pt(s) where $y = 1$.

$\frac{d}{dx}$ $x^8 + 8y^5 = 41y - 32$

$x^8 + 8 = 41 - 32$

$x^8 = 9 - 8$ $\quad x^8 = 1 \quad x = \pm 1$

$8x^7 + 40y^4 \frac{dy}{dx} = 41\frac{dy}{dx}$

$8(1) + 40\frac{dy}{dx} = 41\frac{dy}{dx} \rightarrow \frac{dy}{dx} = 8$ $\qquad y - 1 = 8(x-1)$

and

$8(-1) + 40\frac{dy}{dx} = 41\frac{dy}{dx} \rightarrow \frac{dy}{dx} = -8$ $\qquad y - 1 = -8(x+1)$

Find the derivative

a) $x^3 + 3x^2y + 2xy^2 = 12$

$3x^2 + (-6x)(y) + (-3x^2)\frac{dy}{dx} +$

$(2)(y^2) + (2x)(2y)\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-3x^2 + 6xy - 2y^2}{-3x^2 + 4xy}$$

b) $\sin x + 2\cos 2y = 1$

$\cos x + 2(-\sin(2y))(2\frac{dy}{dx}) = 0$

$$\frac{dy}{dx} = \frac{+\cos x}{+4\sin 2y}$$

Use the table to find the derivatives of the following at $x=4$.

	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
$x=4$	3	-2	7	1
$x=7$	5	-1	4	6

a) $f(x)g(x)$

$f'(4)g(4)+f(4)g'(4)=$

$(-2)(7)+(3)(1) = -11$

b) $\dfrac{f(x)}{g(x)}$

$\dfrac{g(4)f'(4)-f(4)g'(4)}{(g(4))^2}$

$\dfrac{(7)(-2)-(3)(1)}{(7)^2} = \dfrac{-17}{49}$

c) $3f(x)+4g(x)^2$

$3f'(4)+4\cdot 2\cdot g(4)\cdot g'(4)$

$3(-2)+8(7)(1)$

$-6+56$

50

d) $f(g(x))$

$f'(g(4))\cdot g'(4)$

$f'(7)\cdot(1)$

$(-1)\cdot(1)$

-1

Given $s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 5$ is the position function for a particle moving along the x-axis, find the following.

a) displacement from $t=1$ to $t=4$ $s(4)-s(1) = -1.5$

b) times when the particle changes direction $t = 2, 5$ b/c $s'(t)$ changes signs

$s'(t) = t^2 - 7t + 10$ $0 = (t-5)(t-2)$ $+\ \ |\ \ -\ \ |\ \ +$ at 2, 5

c) avg velocity from $t=1$ to $t=4$

$\dfrac{s(4)-s(1)}{4-1} = -.5$

d) instantaneous velocity at $t=3$ $v(3) = s'(3) = 9-21+10 = -2$

e) value of t when the acceleration is 0.

$s''(t) = 2t-7$

$0 = 2t-7$

$t = \dfrac{7}{2}$

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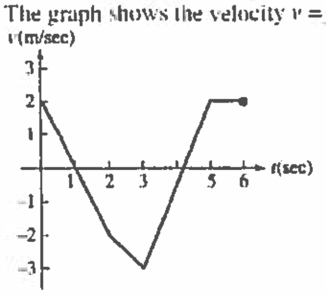
The graph shows the velocity $v = f(t)$ of a particle moving along a horizontal coordinate axis.

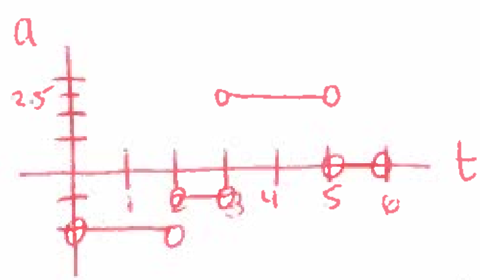


(a) When does the particle reverse direction? $t = 1, 4.2$ $v(t)$ changes signs

(b) When is the particle moving at a constant speed? $(5,6)$

(c) When is the particle moving at its greatest speed? $t = 3$

(d) Graph the acceleration (where defined).



On a test it will explicitly be one or the other → Kinda ambiguous, so I interpreted it 2 ways

Find the derivative

a) $y = \sin^2(\cot(5x))$

$x^2 \to 2x$

$\sin x \to \cos x$

$\cot x \to -\csc^2 x$

$5x \to 5$

$$\frac{dy}{dx} = 2\sin(\cot(5x)) \cdot \cos(\cot(5x)) \cdot -\csc^2(5x) \cdot 5$$

b) ① $\frac{d}{dx}(\sec x)y = x$

$$\sec x \tan x \cdot y + \sec x \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1 - y\sec x \tan x}{\sec x}$$

② or $\frac{d}{dx} \sec(xy) = x$

$$\sec(xy)\tan(xy)\left(1 \cdot y + x\frac{dy}{dx}\right) = 1$$

$$\frac{dy}{dx} = \frac{1 - y\sec(xy)\tan(xy)}{[illegible]}$$