**AB Calculus Differentiation Review** Name: Key

1. Find $\frac{dy}{dx}$

a) $y = 3x^4 + 8x^3 - 1$

$\frac{dy}{dx} = 12x^3 + 24x^2$

b) $y = x^6 \tan x$

$\frac{dy}{dx} = 6x^5 \cdot \tan x + x^6 \cdot \sec^2 x$

c) $y = \frac{7x+6}{8x-6}$

$$\frac{dy}{dx} = \frac{(8x-6)(7) - (7x+6)(8)}{(8x-6)^2}$$

$$\downarrow \quad \frac{56x - 42 - 56x - 48}{(8x-6)^2}$$

$$\downarrow \quad \frac{-90}{(2(4x-3))^2}$$

$$\downarrow \quad \frac{-90}{4(4x-3)^2}$$

$$\downarrow \quad \frac{-45}{2(4x-3)^2}$$

d) $y = \frac{\sqrt{x}-4}{\sqrt{x}+4}$

$$\frac{dy}{dx} = \frac{(\sqrt{x}+4)\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{x}-4)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+4)^2}$$

$$\downarrow \quad \frac{\frac{1}{2} + \frac{2}{\sqrt{x}} - \frac{1}{2} + \frac{2}{\sqrt{x}}}{(\sqrt{x}+4)^2}$$

$$\downarrow \quad \frac{4}{\sqrt{x}\,(\sqrt{x}+4)^2}$$

e) $y = -\frac{6}{x^3} + \frac{6}{\sqrt[6]{x^2}} - \sqrt[4]{x}$

$y = -6x^{-3} + 6x^{-\frac{1}{3}} - x^{\frac{1}{4}}$

$\frac{dy}{dx} = 18x^{-4} - 2x^{-\frac{4}{3}} - \frac{1}{4}x^{-\frac{3}{4}}$

f) $y = \frac{\sin x}{10x}$

$\frac{dy}{dx} = \frac{(10x)(\cos x) - (\sin x)(10)}{(10x)^2}$

$\downarrow$

$\frac{x\cos x - \sin x}{10x^2}$

g) $y = x^5 + \csc x - \cot x$

$\frac{dy}{dx} = 5x^4 - \csc x \cot x + \csc^2 x$

h) $y = \sqrt{6 + \sin 2x}$

$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$

$6 + \sin x \rightarrow \cos x$

$2x \rightarrow 2$

$\frac{dy}{dx} = \frac{1}{2\sqrt{6+\sin 2x}} \cdot \cos 2x \cdot 2$

$= \frac{\cos 2x}{\sqrt{6+\sin 2x}}$

$(\sin x)^5$ $\quad \frac{1}{8}x$

i) $y = \sin^5 x - \sec(10x) + 3\tan\left(\frac{x}{8}\right)$

$\frac{dy}{dx} = 5\sin^4 x \cos x - \sec(10x)\tan(10x) \cdot 10 + 3\sec^2\left(\frac{x}{8}\right) \cdot \frac{1}{8}$

j) $y = (-\sin x)\tan(3x)$

$\frac{dy}{dx} = (-\cos x)\tan(3x) + (-\sin x)(\sec^2(3x)) \cdot 3$

2. If $f(x) = (7x)(\sin x)$ find $f''(x)$.

$f'(x) = 7\sin x + (7x)(\cos x)$

$f''(x) = 7\cos x + 7(\cos x) + (7x)(-\sin x)$

$= 14\cos x - 7x\sin x$

3. The position of a particle moving along the x-axis is given by the function $s(t) = 6\sin t - \cos t$.

a) Find the particle's velocity at time $t = \frac{\pi}{2}$.

$v(t) = 6\cos t + \sin t$ ; $v\left(\frac{\pi}{2}\right) = 0 + 1 = 1$

b) Find the particle's acceleration at time $t = \frac{\pi}{4}$.

$a(t) = -6\sin t + \cos t$

$a\left(\frac{\pi}{4}\right) = -6\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} = -\frac{5\sqrt{2}}{2}$

4. Find the equation of the tangent and normal lines to the equation $f(x) = -9x^2 + 9x$ where $x = 2$.

$f'(x) = -18x + 9$

$f'(2) = -36 + 9$

$= -27$

$f(2) = -9(4) + 9(2)$

$-36 + 18$

$-18$

T: $y + 18 = -27(x - 2)$

N: $y + 18 = \frac{1}{27}(x - 2)$

5. (Calculator) Suppose the cost in dollars of producing x radios is given by the function $C(x) = 400 + 20x - 0.2x^2$. Find the marginal cost when 35 radios are produced.

$C'(x) = 20 - .4x$

$C'(35) = 20 - .4(35) = \$6$ / radio (for the 36th radio)

6. (Calculator) The revenue generated by the sale of x bicycles is given by the function $R(x) = 50x - \frac{x^2}{200}$. Find the additional revenue generated from selling the 501st bicycle.

$\downarrow$

$-\frac{1}{200}x^2$

$R'(x) = 50 - \frac{2}{200}x$

$R'(500) = 50 - \frac{1}{100}(500) = 50 - 5 = \$45$

7. The following table shows the positon of a particle, S, at several times, t, as the particle moves along a straight line, where t is measured in seconds and S is measured in meters. Find the best estimate possible for the velocity of the particle at t = 5 seconds. Indicate units of measure

| t | 3.2 | 4.0 | 6.0 | 7.0 |
|---|---|---|---|---|
| S(t) | 5.7 | 8.1 | 7.5 | 11.2 |

$\frac{7.5 - 8.1}{6 - 4} = \frac{-.6}{2} = -.3$ m/s

8. A particle moves along a line so that its position at any time , $t \geq 0$ is given by the function $s(t) = t^3 - 3t^2 + 3t + 1$ where s is measured in feet and t is measured in seconds. Answer each question with appropriate units.

a) Find the displacement during the first 3 seconds.

$s(3) - s(0) = (27 - 27 + 9 + 1) - (0 + 0 + 0 + 1) = 9$ ft

b) Find the average velocity during the first 3 seconds.

$\frac{s(3) - s(0)}{3 - 0} = \frac{9}{3} = 3$ ft/s

c) Find the instantaneous velocity when t = 3 seconds.

$v(t) = 3t^2 - 6t + 3$

$v(3) = 27 - 18 + 3 = 12$ ft/s

d) Find the acceleration of the particle at t = 3 seconds.

$a(t) = 6t - 6 \quad a(3) = 18 - 6 = 12 \text{ ft/s}^2$

e) Find the speed of the particle when t = 2 seconds.

speed $= |v(t)|$ ; $v(2) = 3(2)^2 - 6(2) + 3 = 3$ ; $|3| = 3$ ft/s

f) At what time(s) is the particle at rest?

$v(t) = 0 \quad 3t^2 - 6t + 3 = 0$; $3(t^2 - 2t + 1) = 0$; $3(t-1)^2 = 0 \quad t = 1$ second

9. (Calculator) The position of a particle at time $t$ seconds, $t \geq 0$, is given by $s(t) = t^4 - 3t^2 + t - 3$, where $t$ is measured in seconds and $s(t)$ is measured in meters. Find the particle's velocity when the acceleration is 0. $v(t) = 4t^3 - 6t + 1 \quad a(t) = 12t^2 - 6 \quad 12t^2 - 6 = 0$

$t^2 = \frac{1}{2}$

$t = \frac{\sqrt{1}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$

$v\left(\frac{\sqrt{2}}{2}\right) \approx -1.828$ m/s

10. If $f(x) = \cos x$, find $f^{(206)}(x)$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f^4(x) = \cos x$

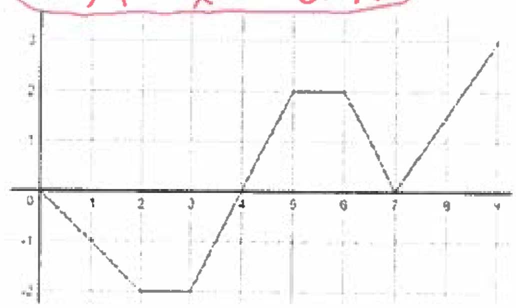
$\frac{206}{4} = 51 \text{ R } 2$

so, $f^{(206)}(x) = -\cos x$

11. $\lim\limits_{h \to 0} \dfrac{\tan\left(\frac{\pi}{4} + h\right) - \tan\left(\frac{\pi}{4}\right)}{h} =$

$\rightarrow \frac{d}{dx} \tan x \rightarrow \sec^2 x$ @ $\pi/4$ $\left(\sec \frac{\pi}{4}\right)^2 \quad \left(\frac{2}{\sqrt{2}}\right)^2 = 2$

12. The graph to the right shows the velocity of a particle moving along the y-axis over a 9 second interval. Use it to answer the following questions for the interval 0 < x < 9 and justify each response.



a) When is the particle speeding up?

when v(t) and a(t) have same sign

$(0,2) \cup (4,5) \cup (7,9)$

b) When is the acceleration of the particle positive?

$(3,5) \cup (7,9)$ Slope of $v(t) > 0$

c) When is the particle moving at a constant speed?

$(2,3) \cup (5,6)$ acceleration = 0

d) What is the particle's speed at time t = 3?

$|-2| = 2 \quad |v(t)| = $ speed

e) When does the particle change direction?

t = 4 (not 7 b/c v(t) doesn't change signs)

f) When is the particle moving up?

$(4,7) \cup (7,9) \quad v(t) > 0$

13. Use the table to find the following derivatives

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 3 | 4 | 2 | $\frac{\pi}{2}$ | $-1$ |
| 4 | 2 | $-3$ | $\frac{1}{4}$ | $-2$ |

a) $\frac{d}{dx}[3f(x)]$ at $x = 3$

b) $\frac{d}{dx}[f(x)g(x)]$ at $x = 4$

c) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ at $x = 4$

d) $\frac{d}{dx}[g(f(x))]$ at $x = 3$

e) $\frac{d}{dx}[\sqrt{f(x)}]$ at $x = 3$

f) $\frac{d}{dx}\left[\frac{1}{g(x)}\right]$ at $x = 3$ $\rightarrow (g(x))^{-1}$

a) $3f'(3) = 3\left(\frac{\pi}{2}\right) = \frac{3\pi}{2}$

b) $f'(4)g(4) + f(4)g'(4)$

$\left(\frac{1}{4}\right)(-3) + (2)(-2)$

$-\frac{3}{4} + (-4) = -4\frac{3}{4}$ or $-\frac{19}{4}$

c) $\frac{g(4)f'(4) - f(4)g'(4)}{(g(4))^2}$

$= \frac{(-3)\left(\frac{1}{4}\right) - (2)(-2)}{(-3)^2}$

$= \frac{-\frac{3}{4} + 4}{9} \rightarrow \frac{3\frac{1}{4}}{9} \rightarrow \frac{\frac{13}{4}}{9} \rightarrow \frac{13}{36}$

d) $g'(f(3)) \cdot f'(3) =$

$g'(4) \cdot \left(\frac{\pi}{2}\right)$

$(-2) \cdot \frac{\pi}{2} = -\pi$

e) $\frac{1}{2\sqrt{f(3)}} \cdot f'(3) =$

$\frac{1}{2\sqrt{4}} \cdot \frac{\pi}{2} = \frac{\pi}{8}$

f) $-1(g(3))^{-2} \cdot g'(3)$

$= \frac{-1}{2^2} \cdot (-1)$

$= \frac{1}{4}$