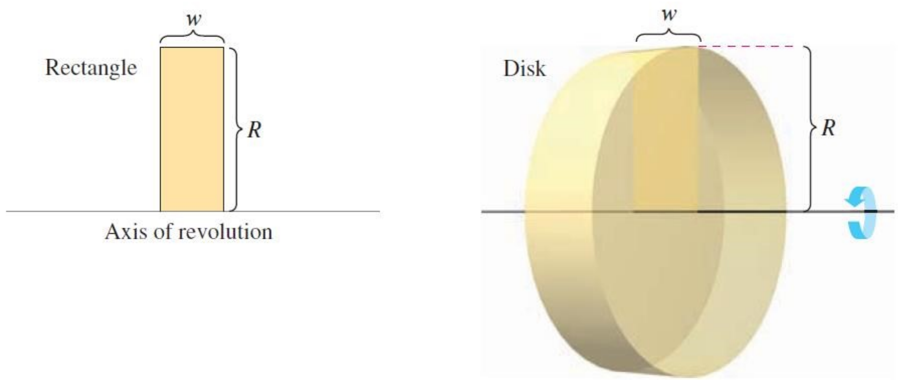
AB Calculus Volume of Solids of Revolution: Disk Method **Name: ___________________________________**

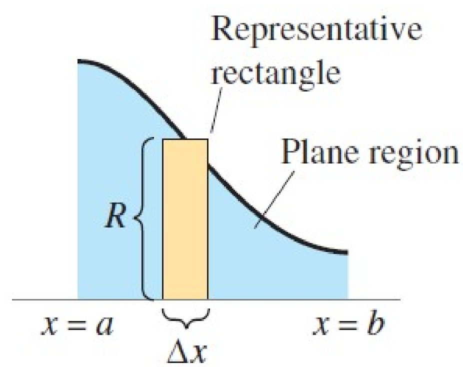
Geogebra Demo to go with this notesheet: https://goo.gl/RmTISe

You have already seen that definite integrals can be used to find area, whether it be underneath a curve or between two curves. Another important application of the definite integral is its use in finding the volume of three-dimensional solids. Today, we will learn how to find the volume of a **solid of revolution**. A solid of revolution is created by revolving a region in a plane about a line. This line is called the **axis of revolution**. The simplest solid that can be created in this manner is a right circular cylinder, or disk, which is formed by rotating a rectangle about an axis that is adjacent to one side of the rectangle.

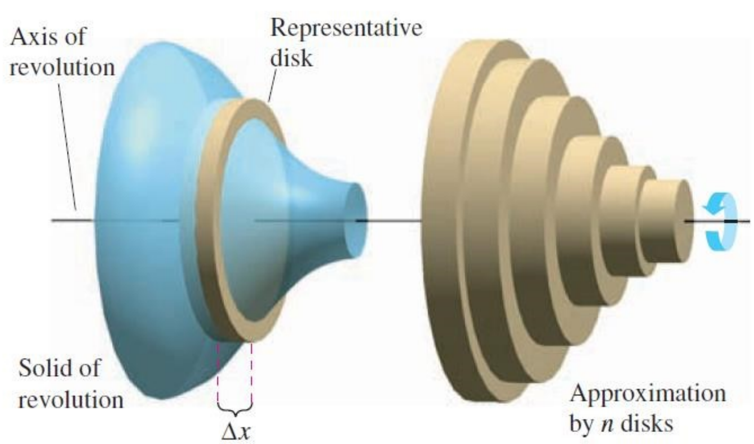


Recall that the formula for finding the volume of a cylinder is $V = \pi r^2 h$. In the picture above, R is the radius of the cylinder and w is the height, so the volume would be $V = \pi R^2 w$.

We can generalize this process to find the volume of any solid of revolution with these characteristics. We are going to start by drawing and shading the region that will be rotated to create the solid. Then, we will draw an arbitrary rectangular strip that represents one rectangle in the Riemann sum approximation. This rectangle will be perpendicular to the axis of revolution. If the rectangle is vertically oriented, we will call it a "dx strip" since the width would be a change in x. if the rectangle is horizontally oriented, we will call it a "dy strip" since the width would be a change in y. An example of a dx strip is shown to the right.



Next, we will find the volume of one cylinder of the solid. We do this in the same manner we did above. The radius of the disk is R and the width, or height, of the cylinder is Δx, or dx. Therefore, we can substitute into the formula for the volume of a cylinder and find the volume of once slice of the solid, which would be $V = \pi R^2 dx$. If we do this process a few times with finite widths we can get an approximation for the volume of the solid, similar to a Riemann sum. Just as before, the more rectangles that are used, the better the approximation. And, just like before, if we find the limit as the number of rectangles approaches infinity, we will get the exact volume of the solid. We can do this by integrating our expression for the volume of one slice of the solid over the interval of the region, or mathematically, $V = \int_a^b \pi R^2 dx$.

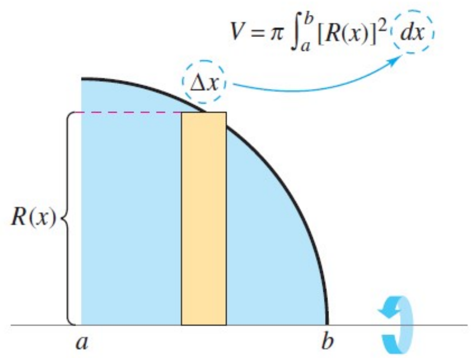


The Disk Method

To find the volume of a solid of revolution with the disk method, use one of the following formulas.

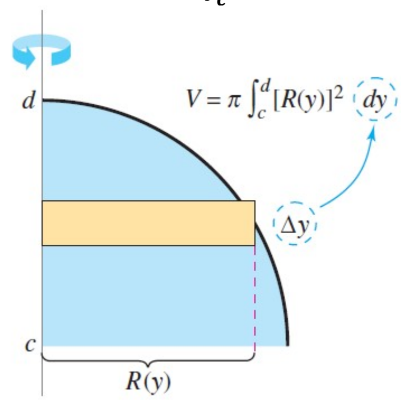
Horizontal Axis of Revolution

$$V = \pi \int_a^b [R(x)]^2 dx$$



Vertical Axis of Revolution

$$V = \pi \int_c^d [R(y)]^2 dy$$



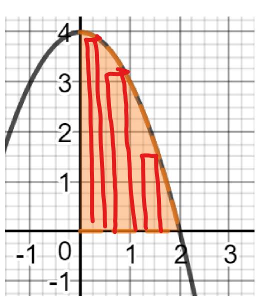
Important: The disk method is only used when the axis of revolution is completely connected to the region.
Important 2: The rectangular strip is always perpendicular to the axis of revolution.

PERPENDISKULAR

Steps to Find the Volume of a Solid Using the Disk Method

1. Draw a picture and shade the desired region.
2. Determine if the axis of revolution is completely connected to the axis of revolution. If it is, proceed.
3. Draw an arbitrary rectangular strip perpendicular to the axis of revolution, representing one rectangle in a Riemann Sum approximation.
4. Find an expression for R, the radius of the disk, by finding an expression for the distance between the outside of the region and the axis of revolution.
5. Find the area of the base of one disk by substituting your expression for R into the formula for the area of a circle, $A = \pi R^2$.
6. Multiply by dx (if vertical strip) or dy (if horizontal strip) to get the volume of one slice of the solid.
7. Integrate over the interval to get the exact volume of the solid of revolution.

Example 1 Find the volume of the solid formed by rotating the region in the first quadrant bounded by graph of $y = 4 - x^2$ and the coordinate axes, around the x-axis.

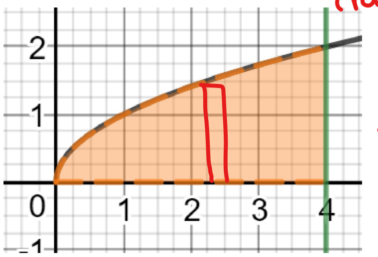


$$V = \pi \int_0^2 (4 - x^2 - 0)^2 \, dx$$

$$= \pi \int_0^2 (4 - x^2)^2 \, dx \approx 53.617$$

(make sure to do things perpendicularly

Example 2 Find the volume of the solid formed by rotating the region bounded by the x-axis, $y = \sqrt{x}$, and $x = 4$ around the x-axis.



R (radius) = height of rectangle

$R = \sqrt{x} - 0 = \sqrt{x}$

$$\text{Volume} = \pi \int_0^4 (\sqrt{x})^2\, dx = \pi \int_0^4 x\, dx = \frac{x^2}{2}(\pi)\Big|_0^4$$

$$= 8\pi$$

Example 3 Find the volume of the solid formed by rotating the region in the first quadrant bounded by the graph of $y = \sqrt{16 - x^2}$ about the y-axis. Vertical Line

(that means my rectangle is horizontal

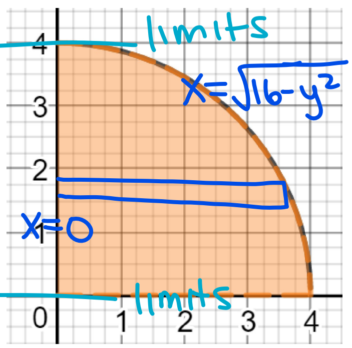
$y = \sqrt{16-x^2}$

$y^2 = 16 - x^2$

$x^2 + y^2 = 16$

$x^2 = 16 - y^2$

$x = \sqrt{16-y^2}$



limits

$x = \sqrt{16-y^2}$

$x = 0$

limits

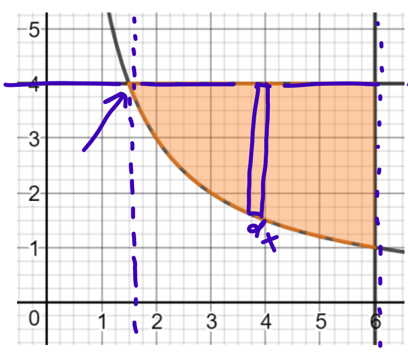
R = right - left (1 to "y")

$R = \sqrt{16-y^2} - 0$

$R = \sqrt{16-y^2}$

$$V = \pi \int_0^4 \left(\sqrt{16-y^2}\right)^2 dy \approx 134.041$$

Example 4 Find the volume of the solid generated by revolving the region bounded by the graphs of $xy = 6$, $y = 4$, and $x = 6$ about the line $y = 4$.



$6/4$

R = top - bottom

$R = 4 - \frac{6}{x}$

$y = \frac{6}{x}$

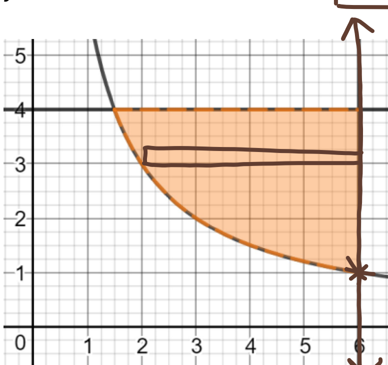
$4 = \frac{6}{x}$

$x = 6/4$

$$V = \pi \int_{6/4}^{6} \left(4 - \frac{6}{x}\right)^2 dx$$

≈ 73.695

Example 5 Find the volume of the solid generated by revolving the region bounded by the graphs of $xy = 6$, $y = 4$, and $x = 6$ about the line $x = 6$.



$x = \frac{6}{y}$

$6 = \frac{6}{y}$

$6y = 6$

$y = 1$

R = Right - Left (1 to "y")

$R = 6 - \frac{6}{y}$

$$V = \pi \int_1^4 \left(6 - \frac{6}{y}\right)^2 dy \approx 110.543$$