Test AB Calculus Name Key

I. Multiple Choice. *Write the correct letter to the left of each question.*

1. Use the graph of the integrand to evaluate $\int_{-6}^{6}\sqrt{36-x^2}\,dx$.

$\frac{1}{2}\pi(6)^2$

18π

A) $\frac{3\pi}{2}$ B) $\frac{9}{4}$ C) $\frac{9\pi}{4}$ D) 18π E) 36π

2. Suppose that $\int_2^3 f(x)dx = 6$ and $\int_2^3 g(x)dx = 5$. Which of the following is not necessarily true?

A) $\int_2^3 3g(x)dx = 15$ ✓ B) $\int_2^3 [f(x)g(x)]dx = 30$ C) $\int_2^3 [f(x)-g(x)]dx = 1$

D) $\int_2^3 [f(x)+g(x)]dx = 11$ E) $\int_2^3 [2f(x)+3g(x)]dx = 27$

3. Choose the correct statement given that $\int_0^7 f(x)dx = 8$ and $\int_1^7 f(x)dx = -3$.

A) $\int_7^1 f(x)dx = -3$ B) $\int_0^1 f(x)dx = 5$ C) $\int_1^0 f(x)dx = 11$

D) $\int_0^1 f(x)dx = 11$ E) $\int_0^1 f(x)dx = -11$

$\int_0^7 = \int_0^1 + \int_1^7$

$8 = \int_0^1 - 3$

4. If $f(x)$ is an even function, then $\int_{-a}^{a} f(x)dx =$

A) $2a$ B) 0 C) $2\int_0^a f(x)dx$ D) Not enough info E) None of these

5. The table below shows the velocity of a person jogging for 24 seconds. Use the right-endpoint values (RRAM) to estimate the distance using 6 intervals of length 4.

Time (sec)	0	4	8	12	16	20	24
Velocity (ft/sec)	0	4	8	12	18	16	14

$4(4+8+12+18+16+14)$

A) 72 ft B) 288 ft C) 144 ft D) 648 ft E) None of the Above

6. Find the average value of the function $g(x) = 2x - 4$ on the interval [1,3].

$\frac{1}{3-1}\int_1^3 2x-4$ $\quad x^2-4x\Big|_1^3$ $\quad (9-12)-(1-4)$ $\quad -3--3$ $\quad \frac{0}{2}=0$

A) –3 B) –6 C) 2 D) 0 E) 6

7. An LRAM sum with 4 equal subdivisions is used to approximate the area under the sine curve from $x = 0$ to $x = \pi$. What is the approximation?

A) $\frac{\pi}{4}\left(0+\frac{\pi}{4}+\frac{\pi}{2}+\frac{3\pi}{4}\right)$ B) $\frac{\pi}{4}\left(0+\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)$ C) $\frac{\pi}{4}\left(0+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}\right)$

D) $\frac{\pi}{4}\left(0+\frac{1}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\right)$ E) $\frac{\pi}{4}\left(\frac{1}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}+1\right)$

$\sin 0 = 0$
$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$\sin \frac{\pi}{2} = 1$

8. A truck moves with positive velocity v(t) from t=3 to t=15. The area under the graph of y= v(t) between 3 and 15 gives

$\int_3^{15} v(t)\,dt = S(15) - S(3)$

A) the velocity of the truck at t= 15.
B) the acceleration of the truck at t= 15.
C) the position of the truck at t= 15.
D) the distance travelled by the truck from t=3 to t= 15.
E) the average position of the truck in the interval from t=3 to t=15.

9. Which of the following quantities would not be represented by the definite integral $\int_0^8 70\,dt$?

$= 70t\Big|_0^8 \rightarrow 70(8) - 70(0)$

A) The distance traveled by a train moving at 70 mph for 8 minutes.
B) The volume of ice cream produced by a machine making 70 gallons per hour for 8 hours.
C) The length of a track left by a snail traveling at 70 cm per hour for 8 hours.
D) The total sales of a company selling $70 of merchandise per hour for 8 hours.
E) The amount the tide has risen 8 minutes after low tide if is rises at a rate of 70 mm per minute during that period.

10. If $\int_a^b f(x)dx = a + 2b$, then $\int_a^b (f(x)+3)dx =$

$\rightarrow a+2b$

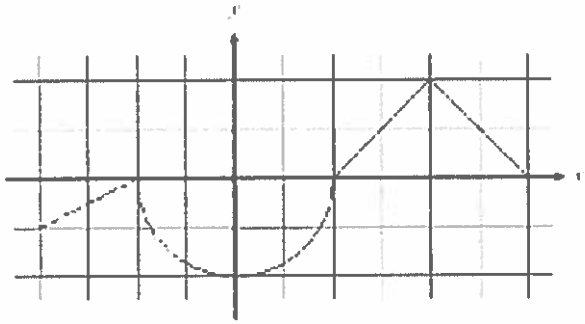
$\int_a^b f(x)\,dx + \int_a^b 3\,dx \rightarrow 3x\Big|_a^b \rightarrow 3b - 3a$

A) $a+2b+3$ B) $3b-3a$ C) $4a-b$ D) $5b-2a$ E) $5b-3a$

$a+2b+3b-3a$
$-2a+5b$

II. Short Answer. *Show all work and box all answers.*

11. The graph of g(x) below consists of line segments and a semicircle. Assume every tick mark is 1 unit.



A) Determine $\int_2^6 g(x)dx$ $4 \cdot 2 \cdot \frac{1}{2} = 4$

B) Determine $\int_{-4}^0 g(x)dx$ $-1 - \frac{1}{4}\pi(4) = -1 - \pi$

C) Determine $\int_3^3 g(x)dx$ $= 0$

D) Determine $\int_2^0 g(x)dx$ $= \pi$

E) Determine the area of the region between the x-axis and the function from x= -4 to x= 0.

$1 + \pi$

Evaluate the following. Use the area of the graphs to determine the integral indicated. Sketch graphs.

12. $\int_{-2}^{4}\left(\frac{x}{2}+5\right)dx$

$\frac{x^2}{4}+5x\Big|_{-2}^{4}$

$\left(\frac{4^2}{4}+5(4)\right)-\left(\frac{4}{4}-10\right)$

$4+20-1+10$

33

13. $\int_{-3}^{4} r\,dr$

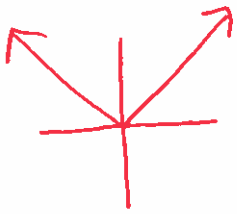
$\frac{r^2}{2}\Big|_{-3}^{4}$

$\frac{1}{2}(16-(-3)^2)$

$\frac{1}{2}(16-9)$

$\frac{1}{2}(7) = 3.5$

14. $\int_{-6}^{8}|x|dx$



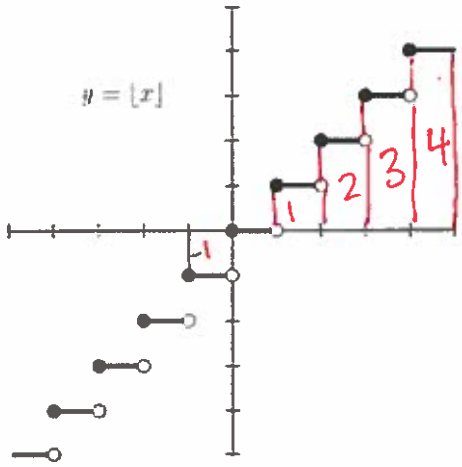
$\frac{1}{2}(6)(6)+\frac{1}{2}(8)(8)$

$18+32$

50

15. $\int_{-1}^{5}\text{int}(x)dx$

9



Test 9 Calculator Part **Name**_____

16. Let "A" be the area under the graph of $f(x)=x^2+3$ from $a=1$ to $b=4$. Approximate A by using left, midpoint, and right rectangle approximation methods. Divide [a,b] into subintervals of equal width $\Delta x=\frac{1}{2}$. Show setup for each part.

1	1.5	2	2.5	3	3.5	4
4	$5\frac{1}{4}$	7	$9\frac{1}{4}$	12	$15\frac{1}{4}$	19

1.25	1.75	2.25	2.75	3.25	3.75

A) Left: $A=\frac{1}{2}\left(4+5\frac{1}{4}+7+9\frac{1}{4}+12+15\frac{1}{4}\right)$

$\frac{1}{2}\left(52\frac{3}{4}\right)=26\frac{3}{8}$

B) Midpoint: $\frac{1}{2}\left(4\frac{9}{16}+6\frac{1}{16}+8\frac{1}{16}+10\frac{9}{16}+13\frac{9}{16}+17\frac{1}{16}\right)$

$=29\frac{15}{16}$

C) Right: $\frac{1}{2}\left(5\frac{1}{4}+7+9\frac{1}{4}+12+15\frac{1}{4}+19\right)$

$\frac{1}{2}\left(67\frac{3}{4}\right)=33\frac{7}{8}$

17. Find the average value of $f(x)=8x^3+2x-1$ on the interval [2,5]. Show the integral setup.

$\frac{1}{5-2}\int_2^5 8x^3+2x-1\,dx$

$\frac{1}{3}\left(2x^4+x^2-x\Big|_2^5\right)$

$\frac{1}{3}\left(2(5)^4+5^2-5-2(2)^4-4+2\right)$

$\frac{1}{3}(1250+25-5-32-2)=\frac{1}{3}(1236)=412$

18. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$$

for $0 \le t \le 30$ where F(t) is measured in cars per minute, and t is measured in minutes.

A) To the nearest whole number, how many total cars pass through the intersection over the 30-minute period? Show integral setup.

$\int_0^{30} 82+4\sin\left(\frac{t}{2}\right)dt \approx 2474$ cars

B) Is the traffic flow increasing or decreasing at t=7? Give a reason for your answer.

↳ depends on derivative

$F'(t) = 4\cos\left(\frac{t}{2}\right)\cdot\frac{1}{2}$

$= 2\cos\left(\frac{t}{2}\right) \to 2\cos 3.5$ → so decreasing

cos in Q3 is neg

sine 3.5 is $\pi < 3.5 < \frac{3\pi}{2} \to Q3$

C) What is the average value of the traffic flow over the time interval from $10 \le t \le 15$? Indicate units of measure and show setup.

$\frac{1}{15-10}\int_{10}^{15} 82+4\sin\left(\frac{t}{2}\right)dt \approx 81.899$ cars/min

D) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure and show setup.

$\frac{F(15)-F(10)}{15-10} = \frac{82+4\sin 7.5 - 82 - 4\sin 5}{5}$

$= 1.517$ cars/min^2