Precalculus Name_____

Dot Products and Unit Vector Notes

Date_____ Period____

Find the dot product of the given vectors.

1) $\mathbf{u} = \langle -7, -9 \rangle$
$\mathbf{v} = \langle 3, -2 \rangle$

$u \cdot v = (-7)(3) + (-9)(-2)$
$= -21 + 18$
$= \boxed{-3}$

2) $\mathbf{u} = \langle 4, -2 \rangle$
$\mathbf{v} = \langle 4, -6 \rangle$

$u \cdot v = (4)(4) + (-2)(-6)$
$= 16 + 12$
$= \boxed{28}$

Find the measure of the angle between the two vectors.

3) $\mathbf{u} = -8\mathbf{i} - 3\mathbf{j}$
$\mathbf{v} = -2\mathbf{i} - 4\mathbf{j}$

42.879°

$u \cdot v = (-8)(-2) + (-3)(-4)$
$= 16 + 12$
$= 28$

$|u| = \sqrt{64+9} = \sqrt{73}$ $|v| = \sqrt{4+16} = \sqrt{20}$

$28 = \sqrt{73}\sqrt{20}\cos\theta$
$\frac{28}{\sqrt{73}\sqrt{20}} = \cos\theta \longrightarrow \cos^{-1}\left(\frac{28}{\sqrt{73}\sqrt{20}}\right) = \theta$

4) $\mathbf{u} = 9\mathbf{i}$
$\mathbf{v} = -6\mathbf{j}$

90°

$u = 9i + 0j$
$v = 0i - 6j$
$u \cdot v = 0 + 0 = 0$

5) $\mathbf{u} = \langle 2, -9 \rangle$
$\mathbf{v} = \langle -4, 18 \rangle$

$(2)(-4) + (-9)(18)$
$-8 - 162$
$-170 = \sqrt{85}\sqrt{340} \cdot \cos\theta$

$\theta = \cos^{-1}\left(\frac{-170}{\sqrt{85}\sqrt{340}}\right)$

$\theta = 180°$ (opposite)

State if the two vectors are parallel, orthogonal, or neither.

6) $\mathbf{u} = 5\mathbf{i} - 35\mathbf{j}$
$\mathbf{v} = -\mathbf{i} + 7\mathbf{j}$

parallel

7) $\mathbf{u} = \frac{3}{2}\mathbf{i} - 3\mathbf{j}$
$\mathbf{v} = 3\mathbf{i} - 6\mathbf{j}$

parallel

8) $\mathbf{u} = 18\mathbf{i} - 18\mathbf{j}$
$\mathbf{v} = 6\mathbf{i} + \mathbf{j}$

$(18)(6)+(-18)(1) \neq 0$

Neither

9) $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$
$\mathbf{v} = -\frac{3}{2}\mathbf{i} + \mathbf{j}$

$2\left(-\frac{3}{2}\right)+3(1) = -3+3 = 0$

orthogonal

Find the component form of the resultant vector.

10) Given: $T = (4, 4)$ $X = (2, 6)$
Unit vector in the direction of $\overline{TX}$

length of 1

$\overrightarrow{TX} = \langle 2-4, 6-4 \rangle$

$\overrightarrow{TX} = \langle -2, 2 \rangle$

*to make a unit vector, you divide by the magnitude

$|\overrightarrow{TX}| = \sqrt{4+4} = \sqrt{8}$

unit vector of $\overrightarrow{TX} = \left\langle \frac{-2}{\sqrt{8}}, \frac{2}{\sqrt{8}} \right\rangle$

11) Given: $P = (-8, -5)$ $Q = (-6, -6)$
Unit vector in the opposite direction of $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \langle -6--8, -6-5 \rangle$

$\langle 2, -1 \rangle$

$\overrightarrow{QP} = \langle -2, 1 \rangle$

$|QP| = \sqrt{4+1} = \sqrt{5}$

unit vector $= \left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

Express the resultant vector as a linear combination of unit vectors i and j.

12) $|\mathbf{f}| = 4, 198°$
Unit vector in the direction of **f**

$\vec{f} = 4\cos 198\,i + 4\sin 198\,j$

unit vector $= \cos 198\,i + \sin 198\,j$

$= -.951\,i - .309\,j$

13) $|\mathbf{u}| = 22, 115°$
Unit vector in the direction of **u**

$\vec{u} = 22\cos 115°\,i + 22\sin 115\,j$

unit vector $= \cos 115°\,i + \sin 115°\,j$

$= -.423\,i + .906\,j$

Find the magnitude and direction angle of the resultant vector.

14) Given: $T = (5, 7)$ $X = (-5, 6)$
$Y = (6, -5)$ $Z = (-7, 6)$
Find: $\overrightarrow{TX} - \overrightarrow{YZ}$

$\overrightarrow{TX} = \langle -5-5, 6-7 \rangle$

$= \langle -10, -1 \rangle$

$\overrightarrow{YZ} = \langle -7-6, 6--5 \rangle$

$= \langle -13, 11 \rangle$

$\langle -10, -1 \rangle - \langle -13, 11 \rangle = \langle 3, -12 \rangle$

mag $\sqrt{9+144} = \sqrt{153}$

$\tan^{-1}\left(\frac{12}{3}\right) = 75.964°$ S of E or $284.036°$

15) Given: $P = (0, 7)$ $Q = (4, 2)$
Unit vector in the opposite direction of $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \langle 4-0, 2-7 \rangle$

$= \langle 4, -5 \rangle$

Mag 1 (it's a unit vector)

angle $\tan^{-1}\left(\frac{5}{4}\right) \rightarrow 51.340°$

N of W (b/c it said opposite) or $128.66°$

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–3

2) $\mathbf{u} = \langle 4, -2 \rangle$
$\mathbf{v} = \langle 4, -6 \rangle$

28

Find the measure of the angle between the two vectors.

3) $\mathbf{u} = -8\mathbf{i} - 3\mathbf{j}$
$\mathbf{v} = -2\mathbf{i} - 4\mathbf{j}$

42.88°

4) $\mathbf{u} = 9\mathbf{i}$
$\mathbf{v} = -6\mathbf{j}$

90°

5) $\mathbf{u} = \langle 2, -9 \rangle$
$\mathbf{v} = \langle -4, 18 \rangle$

180°

State if the two vectors are parallel, orthogonal, or neither.

6) $\mathbf{u} = 5\mathbf{i} - 35\mathbf{j}$
$\mathbf{v} = -\mathbf{i} + 7\mathbf{j}$

Parallel

7) $\mathbf{u} = \frac{3}{2}\mathbf{i} - 3\mathbf{j}$
$\mathbf{v} = 3\mathbf{i} - 6\mathbf{j}$

Parallel

8) $\mathbf{u} = 18\mathbf{i} - 18\mathbf{j}$
$\mathbf{v} = 6\mathbf{i} + \mathbf{j}$

Neither

9) $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$
$\mathbf{v} = -\frac{3}{2}\mathbf{i} + \mathbf{j}$

Orthogonal

Find the component form of the resultant vector.

10) Given: $T = (4, 4)$ $X = (2, 6)$
Unit vector in the direction of $\overrightarrow{TX}$

$\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

11) Given: $P = (-8, -5)$ $Q = (-6, -6)$
Unit vector in the opposite direction of $\overrightarrow{PQ}$

$\left\langle -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$

Express the resultant vector as a linear combination of unit vectors i and j.

12) $|\mathbf{f}| = 4, 198°$
Unit vector in the direction of **f**

$-0.95\mathbf{i} - 0.31\mathbf{j}$

13) $|\mathbf{u}| = 22, 115°$
Unit vector in the direction of **u**

$-0.42\mathbf{i} + 0.91\mathbf{j}$

Find the magnitude and direction angle of the resultant vector.

14) Given: $T = (5, 7)$ $X = (-5, 6)$
$Y = (6, -5)$ $Z = (-7, 6)$
Find: $\overrightarrow{TX} - \overrightarrow{YZ}$

$3\sqrt{17} \approx 12.369; 284.04°$

15) Given: $P = (0, 7)$ $Q = (4, 2)$
Unit vector in the opposite direction of $\overrightarrow{PQ}$

$128.66°$